Clarification of Standards for Parents
Grade 4 Mathematics Unit 1

Dear Parents,
We want to make sure that you have an understanding of the mathematics your child will be learning this year. Below you will find the standards we will be learning in Unit One. Each standard is in bold print and underlined and below it is an explanation with student examples. Your child is not learning math the way we did when we were in school, so hopefully this will assist you when you help your child at home. Please let your teacher know if you have any questions. ☺

MGSE.4.NBT.1 Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*
This standard calls for students to extend their understanding of place value related to multiplying and dividing by multiples of 10. In this standard, students should reason about the magnitude of digits in a number. Students should be given opportunities to reason and analyze the relationships of numbers that they are working with.

Example:
How is the 2 in the number 582 similar to and different from the 2 in the number 528?

MGSE4.NBT.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
This standard refers to various ways to write numbers. Students should have flexibility with the different number forms. Traditional expanded form is 285 = 200 + 80 + 5. Written form is two hundred eighty-five. However, students should have opportunities to explore the idea that 285 could also be 28 tens plus 5 ones or 1 hundred, 18 tens, and 5 ones. Students should also be able to compare two multi-digit whole numbers using appropriate symbols.

Common Misconceptions:
There are several misconceptions students may have about writing numerals from verbal descriptions. Numbers like one thousand two causes problems for students. Many students will understand the 1000 and the 2 but instead of placing the 2 in the ones place, students will write the numbers as they can hear them, 10002 (ten thousand two). There are multiple strategies that can be used to assist with this concept, including place-value boxes and vertical-addition methods.

Students often assume that the first digit of a multi-digit number indicates the “greatness” of a number. The assumption is made the 954 is greater than 1002 because students are focusing on the first digit instead of the number as a whole.

Students need to be aware of the greatest place value. In this example, there is one number with the lead digit in the thousands and another numbers with its lead digit in the hundreds.

Development of a clear understanding of the value of the digits in a number is critical for the understanding of and using numbers in computations. Helping students build the understanding that 12345 means one ten thousand or 10,000, two thousands or 2000, three hundreds or 300, four tens or 40, and 5 ones or 5. Additionally, the answer is the sum of each of these values 10,000 + 2000 + 300 + 40 + 5.

MGSE4.NBT.3 Use place value understanding to round multi-digit whole numbers to any place.
This standard refers to place value understanding, which extends beyond an algorithm or procedure for rounding. The expectation is that students have a deep understanding of place value and number sense and can explain and reason about the answers they get when they round. Students should have numerous experiences using a number line and a hundreds chart as tools to support their work with rounding.

Example:
On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many total miles did your family travel?
Some typical estimation strategies for this problem:

Student 1	**Student 2**	**Student 3**
I first thought about 276 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500.	I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundreds that I already had, I end up with 500.	I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200, and 30, I know my answer will about 530.

Example:
Round 368 to the nearest hundred.

This will either be 300 or 400, since those are the two hundreds before and after 368. Draw a number line, subdivide it as much as necessary, and determine whether 368 is closer to 300 or 400. Since 368 is closer to 400, this number should be rounded to 400.

MGSE4.NBT.4 Fluently add and subtract multi-digit whole numbers using the standard algorithm.
Students build on their understanding of addition and subtraction, their use of place value and their flexibility with multiple strategies to make sense of the standard algorithm. They continue to use place value in describing and justifying the processes they use to add and subtract.

This standard refers to fluency, which means accuracy, efficiency (using a reasonable amount of steps and time), and flexibility (using a variety strategies such as the distributive property). This is the first grade level in which students are expected to be proficient at using the standard algorithm to add and subtract. However, other previously learned strategies are still appropriate for students to use.

When students begin using the standard algorithm their explanation may be quite lengthy. After much practice with using place value to justify their steps, they will develop fluency with the algorithm. Students should be able to explain why the algorithm works.

Example:
$$\begin{array}{r} 3892 \\ \underline{+\ 1567} \end{array}$$

Student explanation for this problem:
1. Two ones plus seven ones is nine ones.
2. Nine tens plus six tens is 15 tens.
3. I am going to write down five tens and think of the 10 tens as one more hundred. (*Denotes with a 1 above the hundreds column*)
4. Eight hundreds plus five hundreds plus the extra hundred from adding the tens is 14 hundreds.

5. I am going to write the four hundreds and think of the 10 hundreds as one more 1000. (*Denotes with a 1 above the thousands column*)
6. Three thousands plus one thousand plus the extra thousand from the hundreds is five thousand.

Example:
$$\begin{array}{r} 3546 \\ \underline{-\ 928} \end{array}$$

Student explanations for this problem:

1. There are not enough ones to take 8 ones from 6 ones so I have to use one ten as 10 ones. Now I have 3 tens and 16 ones. (*Marks through the 4 and notates with a 3 above the 4 and writes a 1 above the ones column to be represented as 16 ones.*)
2. Sixteen ones minus 8 ones is 8 ones. (*Writes an 8 in the ones column of answer.*)
3. Three tens minus 2 tens is one ten. (*Writes a 1 in the tens column of answer.*)
4. There are not enough hundreds to take 9 hundreds from 5 hundreds so I have to use one thousand as 10 hundreds. (*Marks through the 3 and notates with a 2 above it. Writes down a 1 above the hundreds column.*) Now I have 2 thousand and 15 hundreds.
5. Fifteen hundreds minus 9 hundreds is 6 hundreds. (*Writes a 6 in the hundreds column of the answer*.)
6. I have 2 thousands left since I did not have to take away any thousands. (*Writes 2 in the thousands place of answer.*)

Students should know that it is mathematically possible to subtract a larger number from a smaller number but that their work with whole numbers does not allow this as the difference would result in a negative number.

Common Misconceptions

Often students mix up when to "carry" and when to "borrow". Also students often do not notice the need of borrowing and just take the smaller digit from the larger one. Emphasize place value and the meaning of the digits.

If students are having difficulty with linking up similar place values in numbers as they are adding and subtracting, it is sometimes helpful to have them write their calculations on the grid paper. This assists the student with lining up the numbers more accurately.

If students are having a difficult time with a standard addition algorithm, a possible modification to the algorithm might be helpful. Instead of the "shorthand" of "carrying," students could add by place value, moving left to right placing the answers down below the "equals" line. For example:

$$\begin{array}{rl} 249 & \\ \underline{+\ 372} & \\ 500 & \text{(Start with 200 + 300 to get the 500} \\ 110 & \text{then 40 + 70 to get 110} \\ \underline{11} & \text{and 9 + 2 to get 11.)} \\ 621 & \end{array}$$

MGSE4.OA.3 Solve multistep word problems with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a symbol or letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

The focus in this standard is to have students use and discuss various strategies. It refers to estimation strategies, including using compatible numbers (numbers that sum to 10 or 100) or rounding. Problems should be structured so that all acceptable estimation strategies will arrive at a reasonable answer. Students need many opportunities solving multistep story problems.

In unit one, students focus on solving multistep word problems with addition and subtraction.

Example 1:
On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. About how many miles did they travel total?

Some typical estimation strategies for this problem are shown below.

Student 1	Student 2	Student 3
I first thought about 267 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500.	I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundreds that I already had, I end up with 500.	I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200, and 30, I know my answer will be about 530.

The assessment of estimation strategies should only have one reasonable answer (500 or 530), or a range (between 500 and 550). Problems will be structured so that all acceptable estimation strategies will arrive at a reasonable answer.

MGSE4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
This standard includes multi-step word problems related to expressing measurements from a larger unit in terms of a smaller unit (e.g., feet to inches, meters to centimeter, dollars to cents). Students should have ample opportunities to use number line diagrams to solve word problems.

In unit one, students focus on solving measurement word problems that involve the operations of addition and subtraction. Students also use diagrams (such as number line diagrams) to solve problems.

Example:
Mason ran for an hour and 15 minutes on Monday, 25 minutes on Tuesday, and 40 minutes on Wednesday. What was the total number of minutes Mason ran?

Students can add the times to find the total number of minutes Mason ran. 40 minutes plus another 25 minutes would be 65 minutes, or an hour and 5 minutes. Then, an hour and five minutes can be added to an hour and 15 minutes to see that Mason ran 2 hours and 20 minutes in all.

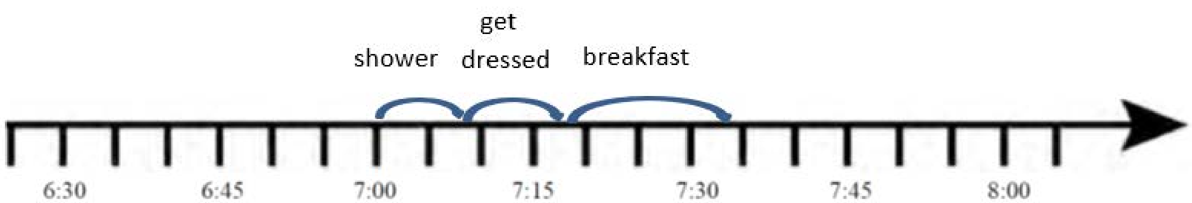
Example:
A pound of apples costs $1.20. Rachel bought a pound and a half of apples. If she gave the clerk a $5.00 bill, how much change will she get back?

Possible student solution: If Rachel bought a pound and a half of apples, she paid $1.20 for the first pound and then 60¢ for the other half a pound, since half of $1.20 is 60¢. When I add $1.20 and 60¢, I get a total of $1.80 spent on the apples. If she gave the clerk a five dollar bill, I can count up to find out how much change she received.

$1.80 + 20¢ = $2.00
$2.00 + 3.00 = $5.00
$3.00 + 20¢ = $3.20
So Rachel got $3.20 back in change.

Number line diagrams that feature a measurement scale can represent measurement quantities. Examples include: ruler, diagram marking off distance along a road with cities at various points, a timetable showing hours throughout the day, or a liquid volume measure on the side of a container.

Example:
At 7:00 a.m. Candace wakes up to go to school. It takes her 8 minutes to shower, 9 minutes to get dressed and 17 minutes to eat breakfast. How many minutes does she have until the bus comes at 8:00 a.m.? Use the number line to help solve the problem.



Candace is finished at 7:34. If the bus comes at 8:00, I can count on to from 7:34 to 8:00 to find how many minutes it takes for the bus to arrive. From 7:34 to 7:35 is one minute. From 7:35 to 7:40 is 5 minutes and from 7:40 to 8:00 is 20 minutes. 1 minute + 5 minutes + 20 minutes = 26 minutes until the bus arrives.

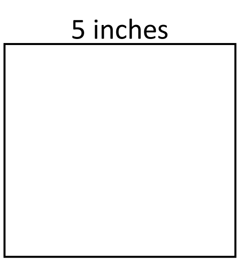
MGSE4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*

In unit one, students focus on applying the perimeter formulas for rectangles in real world and mathematical problems.

Students developed understanding of perimeter in 3^{rd} grade by using visual models.

While students are expected to use formulas to calculate perimeter of rectangles, they need to understand and be able to communicate their understanding of why the formulas work. The formula for perimeter can be $2l + 2w$ or $2(l + w)$ and the answer will be in linear units. This standard calls for students to generalize their understanding of perimeter by connecting the concepts to mathematical formulas. These formulas should be developed through experience, not just memorization.

Example:
Find the perimeter of the square:



Since a square has four sides that are equal in length, each side must measure 5 inches. Using the perimeter formulas:
$2l + 2w = (2 \times 5) + (2 \times 5) = 10 + 10 = 20$ inches

$2(l + w) = 2(2 \times 5) = 2(10) = 20$ inches

The perimeter of the square is 20 inches.

(Adapted from Henry County Schools)

<u>Clarification of Standards for Parents</u>
<u>Grade 4 Mathematics Unit 2</u>

Dear Parents,

We want to make sure that you have an understanding of the mathematics your child will be learning this year. Below you will find the standards we will be learning in Unit Two. Each standard is in bold print and underlined and below it is an explanation with student examples. Your child is not learning math the way we did when we were in school, so hopefully this will assist you when you help your child at home. Please let your teacher know if you have any questions. ☺

<u>MGSE4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.</u>
A *multiplicative comparison* is a situation in which one quantity is multiplied by a specified number to get another quantity (e.g., "*a* is *n* times as much as *b*"). Students should be able to identify and verbalize which quantity is being multiplied and which number tells how many times.

Students should be given opportunities to write and identify equations and statements for multiplicative comparisons.

Examples:
5 x 8 = 40: Sally is five years old. Her mom is eight times older. How old is Sally's Mom?
5 x 5 = 25: Sally has five times as many pencils as Mary. If Sally has 5 pencils, how many does Mary have?

<u>MGSE4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.</u>
This standard calls for students to translate comparative situations into equations with an unknown and solve.

Examples:

- **Unknown Product:** A blue scarf costs $3. A red scarf costs 6 times as much. How much does the red scarf cost? $(3 \times 6 = p)$
- **Group Size Unknown:** A book costs $18. That is 3 times more than a DVD. How much does a DVD cost? $(18 \div p = 3 \text{ or } 3 \times p = 18)$
- **Number of Groups Unknown:** A red scarf costs $18. A blue scarf costs $6. How many times as much does the red scarf cost compared to the blue scarf? $(18 \div 6 = p \text{ or } 6 \times p = 18)$
  When distinguishing multiplicative comparison from additive comparison, students should note the following.
- Additive comparisons focus on the difference between two quantities.
  - For example, Deb has 3 apples and Karen has 5 apples. How many more apples does Karen have?
  - A simple way to remember this is, "How many more?"
- Multiplicative comparisons focus on comparing two quantities by showing that one quantity is a specified number of times larger or smaller than the other.
  - For example, Deb ran 3 miles. Karen ran 5 times as many miles as Deb. How many miles did Karen run?
  A simple way to remember this is "How many times as much?" or "How many times as many?"

<u>MGSE4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.</u>
The focus in this standard is to have students use and discuss various strategies. It refers to estimation strategies, including using compatible numbers (numbers that sum to 10 or 100) or rounding. Problems should be structured so that all acceptable estimation strategies will arrive at a reasonable answer.

Example 1:
On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many miles did they travel total?

Some typical estimation strategies for this problem are shown below.

Student 1	**Student 2**	**Student 3**
I first thought about 267 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500.	I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundreds that I already had, I end up with 500.	I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200, and 30, I know my answer will be about 530.

The assessment of estimation strategies should only have one reasonable answer (500 or 530), or a range (between 500 and 550). Problems will be structured so that all acceptable estimation strategies will arrive at a reasonable answer.

Example 2:
Your class is collecting bottled water for a service project. The goal is to collect 300 bottles of water. On the first day, Max brings in 3 packs with 6 bottles in each container. Sarah wheels in 6 packs with 6 bottles in each container. About how many bottles of water still need to be collected?

Student 1	**Student 2**
First I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 plus 36 is about 50. I'm trying to get to 300. 50 plus another 50 is 100. Then I need 2 more hundreds. So we still need 250 bottles.	First I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 is about 20 and 36 is about 40. 40 + 20 = 60. 300 – 60 = 240, so we need about 240 more bottles.

This standard references interpreting remainders. Remainders should be put into context for interpretation. Ways to address remainders:

- Remain as a left over
- Partitioned into fractions or decimals
- Discarded leaving only the whole number answer
- Increase the whole number answer up one
- Round to the nearest whole number for an approximate result

Example:
Write different word problems involving **44 ÷ 6 = ?** where the answers are best represented as:

- Problem A: 7
- Problem B: 7 r 2
- Problem C: 8
- Problem D: 7 or 8
- Problem E: $7\ {}^{2}/_{6}$

Possible solutions:

- Problem **A: 7.**
  Mary had 44 pencils. Six pencils fit into each of her pencil pouches. How many pouches did she fill? 44 ÷ *6 = p; p = 7 r 2. Mary can fill 7 pouches completely.*

- Problem **B: 7 r 2.**
  Mary had 44 pencils. Six pencils fit into each of her pencil pouches. How many pouches could she fill and how many pencils would she have left? 44 ÷ *6 = p; p = 7 r 2; Mary can fill 7 pouches and have 2 left over.*

- Problem **C: 8.**
  Mary had 44 pencils. Six pencils fit into each of her pencil pouches. What would the fewest number of pouches she would need in order to hold all of her pencils? 44 ÷ *6 = p; p = 7 r 2; Mary can needs 8 pouches to hold all of the pencils.*

- Problem **D: 7 or 8.**
  Mary had 44 pencils. She divided them equally among her friends before giving one of the leftovers to each of her friends. How many pencils could her friends have received? 44 ÷ *6 = p; p = 7 r 2; Some of her friends received 7 pencils. Two friends received 8 pencils.*

- Problem **E: $7^2/_6$.**
  Mary had 44 pencils and put six pencils in each pouch. What fraction represents the number of pouches that Mary filled? 44 ÷ *6 = p; p =* $7^2/_6$
  Example:
  There are 128 students going on a field trip. If each bus held 30 students, how many buses are needed? *(128 ÷ 30 = b; b = 4 R 8; They will need 5 buses because 4 busses would not hold all of the students).*

Students need to realize in problems, such as the example above, that an extra bus is needed for the 8 students that are left over. Estimation skills include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of situations using various estimation strategies. Estimation strategies include, but are not limited to the following.

- **Front-end estimation with adjusting** (Using the highest place value and estimating from the front end, making adjustments to the estimate by taking into account the remaining amounts)
- **Clustering around an average** (When the values are close together an average value is selected and multiplied by the number of values to determine an estimate.)
- **Rounding and adjusting** (Students round down or round up and then adjust their estimate depending on how much the rounding affected the original values.)
- **Using friendly or compatible numbers such as factors** (Students seek to fit numbers together; e.g., rounding to factors and grouping numbers together that have round sums like 100 or 1000.)
- **Using benchmark numbers that are easy to compute** (Students select close whole numbers for fractions or decimals to determine an estimate.)

MGSE4.OA.4 Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

This standard requires students to demonstrate understanding of factors and multiples of whole numbers. This standard also refers to prime and composite numbers. Prime numbers have exactly two factors, the number one and their own number. For example, the number 17 has the factors of 1 and 17. Composite numbers have more than two factors. For example, 8 has the factors 1, 2, 4, and 8.

Common Misconceptions
A common misconception is that the number 1 is prime, when in fact; it is neither prime nor composite. Another common misconception is that all prime numbers are odd numbers. This is not true, since the number 2 has only 2 factors, 1 and 2, and is also an even number.

When listing multiples of numbers, students may not list the number itself. Emphasize that the smallest multiple is the number itself.

Some students may think that larger numbers have more factors. Having students share all factor pairs and how they found them will clear up this misconception.

Prime vs. Composite:

- A prime number is a number greater than 1 that has only 2 factors, 1 and itself.
- Composite numbers have more than 2 factors.
- Students investigate whether numbers are prime or composite by building rectangles (arrays) with the given area and finding which numbers have more than two rectangles (e.g., 7 can be made into only 2 rectangles, 1 × 7 and 7 × 1, therefore it is a prime number).
- Finding factors of the number. Students should understand the process of finding factor pairs so they can do this for any number 1-100.

Example:
Factor pairs for 96: 1 and 96, 2 and 48, 3 and 32, 4 and 24, 6 and 16, 8 and 12.
Multiples can be thought of as the result of skip counting by each of the factors. When skip counting, students should be able to identify the number of factors counted e.g., 5, 10, 15, 20 (there are 4 fives in 20).

Example:
Factors of 24: 1, 2, 3, 4, 6,8, 12, 24
Multiples: 1, 2, 3, 4, 5, … , <u>24</u>
2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, <u>24</u>
3, 6, 9, 12, 15, 15, 21, <u>24</u>
4, 8, 12, 16, 20, <u>24</u>
8, 16, <u>24</u>
12, <u>24</u>
<u>24</u>

To determine if a number between 1-100 is a multiple of a given one-digit number, some helpful hints include the following:

- All even numbers are multiples of 2.
- All even numbers that can be halved twice (with a whole number result) are multiples of 4.
- All numbers ending in 0 or 5 are multiples of 5.

<u>**MGSE4.OA.5 Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.** ***For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.***</u>
Patterns involving numbers or symbols either repeat or grow. Students need multiple opportunities creating and extending number and shape patterns. Numerical patterns allow students to reinforce facts and develop fluency with operations.

Patterns and rules are related. A pattern is a sequence that repeats the same process over and over. A rule dictates what that process will look like. Students investigate different patterns to find rules, identify features in the patterns, and justify the reason for those features.

Example

Pattern	Rule	Feature(s)
3, 8, 13, 18, 23,28, .	Start with 3; add 5	The numbers alternately end with a 3 or an 8
5, 10, 15, 20, …	Start with 5; add 5	The numbers are multiples of 5 and end with either 0 or 5. The numbers that 3nd with 5 are products of 5 and an odd number. The numbers that end in 0 are products of 5 and an even number.

After students have identified rules and features from patterns, they need to generate a numerical or shape pattern from a given rule.

Example:
Rule: Starting at 1, create a pattern that starts at 1 and multiplies each number by 3. Stop when you have 6 numbers.

Students write 1, 3, 9, 27, 8, 243. Students notice that all the numbers are odd and that the sums of the digits of the 2 digit numbers are each 9. Some students might investigate this beyond 6 numbers. Another feature to investigate is the patterns in the differences of the numbers (3 – 1 = 2, 9 – 3 = 6, 27 – 9 = 18, etc.).

This standard calls for students to describe features of an arithmetic number pattern or shape pattern by identifying the rule, and features that are not explicit in the rule. A t-chart is a tool to help students see number patterns.

Example:
There are 4 beans in the jar. Each day 3 beans are added. How many beans are in the jar for each of the first 5 days?

Day	Operation	Beans
0	$3 \times 0 + 4$	4
1	$3 \times 1 + 4$	7
2	$3 \times 2 + 4$	10
3	$3 \times 3 + 4$	13
4	$3 \times 4 + 4$	16
5	$3 \times 5 + 4$	19

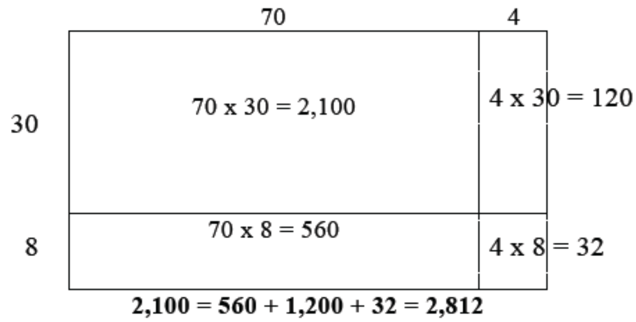
MGSE4.NBT.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
Students who develop flexibility in breaking numbers apart have a better understanding of the importance of place value and the distributive property in multi-digit multiplication. Students use base ten blocks, area models, partitioning, compensation strategies, etc. when multiplying whole numbers and use words and diagrams to explain their thinking. They use the terms factor and product when communicating their reasoning. Multiple strategies enable students to develop fluency with multiplication and transfer that understanding to division. **Use of the standard algorithm for multiplication is an expectation in the fifth grade.**

This standard calls for students to multiply numbers using a variety of strategies.

Example:
There are 25 dozen cookies in the bakery. What is the total number of cookies at the bakery?

Student 1	Student 2	Student 3
25×12 I broke 12 up into 10 and 2. $25 \times 10 = 250$ $25 \times 2 = 50$ $250 + 50 = 300$	25×12 I broke 25 into 5 groups of 5. $5 \times 12 = 60$ I have 5 groups of 5 in 25. $60 \times 5 = 300$	25×12 I doubled 25 and cut 12 in half to get 50×6. $50 \times 6 = 300$

Example:
What would an array area model of 74 x 38 look like?

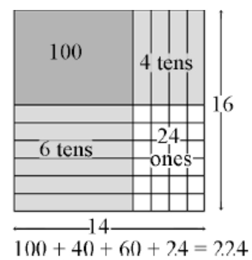


Examples:
To illustrate 154 × 6, students use base 10 blocks or use drawings to show 154 six times. Seeing 154 six times will lead them to understand the distributive property,

$154 \times 6 = (100 + 50 + 4) \times 6$
$= (100 \times 6) + (50 \times 6) + (4 \times 6)$
$=600 + 300 + 24 = 924.$

The area model below shows the partial products for 14 x 16 = 224.
Using the area model, students first verbalize their understanding:

- 10×10 is 100
- 4×10 is 40
- 10×6 is 60, and
- 4×6 is 24.



Students use different strategies to record this type of thinking.

Students explain this strategy and the one below with base 10 blocks, drawings, or numbers.

  25
<u>× 24</u>
 400 (20 × 20)
 100 (20 × 5)
 80 (4 × 20)
 <u>20</u> (4 × 5)
600

<u>MGSE.4.NBT.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</u>
In fourth grade, students build on their third grade work with division within 100. Students need opportunities to develop their understandings by using problems in and out of context.

Example:
A 4th grade teacher bought 4 new pencil boxes. She has 260 pencils. She wants to put the pencils in the boxes so that each box has the same number of pencils. How many pencils will there be in each box?

- **Using Base 10 Blocks:** Students build 260 with base 10 blocks and distribute them into 4 equal groups. Some students may need to trade the 2 hundreds for tens but others may easily recognize that 200 divided by 4 is 50.
- **Using Place Value:** $260 \div 4 = (200 \div 4) + (60 \div 4)$
- **Using Multiplication:** $4 \times 50 = 200$, $4 \times 10 = 40$, $4 \times 5 = 20$; $50 + 10 + 5 = 65$; so $260 \div 4 = 65$

This standard calls for students to explore division through various strategies.

Example:

There are 592 students participating in Field Day. They are put into teams of 8 for the competition. How many teams get created?

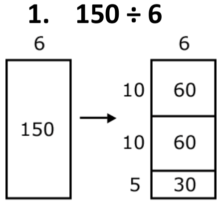
Student 1	**Student 2**	**Student 3**
592 divided by 8 There are 70 eights in 560. 592 – 560 = 32 There are 4 eights in 32. 70 + 4 = 74	592 divided by 8 I know that 10 eights is 80. If I take out 50 eights that is 400. 592 – 400 = 192 I can take out 20 more eights which is 160. 192 – 160 = 32 8 goes into 32 four times. I have none left. I took out 50, then 20 more, then 4 more. That's 74.	I want to get to 592. $8 \times 25 = 200$ $8 \times 25 = 200$ $8 \times 25 = 200$ 200 + 200 + 200 = 600 600 – 8 = 592 I had 75 groups of 8 and took one away, so there are 74 teams.

Example:

Using an Open Array or Area Model

After developing an understanding of using arrays to divide, students begin to use a more abstract model for division. This model connects to a recording process that will be formalized in the 5th grade.

1. 150 ÷ 6



Students make a rectangle and write 6 on one of its sides. They express their understanding that they need to think of the rectangle as representing a total of 150.

1. Students think, "6 times what number is a number close to 150?" They recognize that 6×10 is 60 so they record 10 as a factor and partition the rectangle into 2 rectangles and label the area aligned to the factor of 10 with 60. They express that they have only used 60 of the 150 so they have 90 left.
2. Recognizing that there is another 60 in what is left, they repeat the process above. They express that they have used 120 of the 150 so they have 30 left.
3. Knowing that 6×5 is 30, they write 30 in the bottom area of the rectangle and record 5 as a factor.
4. Student express their calculations in various ways:
   a.
   $$\begin{array}{rl} 150 & \\ \underline{-60} & (6 \times 10) \\ 90 & \\ \underline{-60} & (6 \times 10) \\ 30 & \\ \underline{-30} & (6 \times 5) \\ 0 & \end{array}$$
   $150 \div 6 = 10 + 10 + 5 = 25$

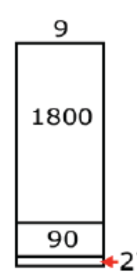
   b. $150 \div 6 = (60 \div 6) + (60 \div 6) + (30 \div 6) = 10 + 10 + 5 = 25$

Example:

1917 ÷ 9

A student's description of his or her thinking may be:

I need to find out how many 9s are in 1917. I know that 200 x 9 is 1800. So if I use 1800 of the 1917, I have 117 left. I know that 9 x 10 is 90. So if I have 10 more 9s, I will have 27 left. I can make 3 more 9s. I have 200 nines, 10 nines and 3 nines. So I made 213 nines. 1917 ÷ 9 = 213.



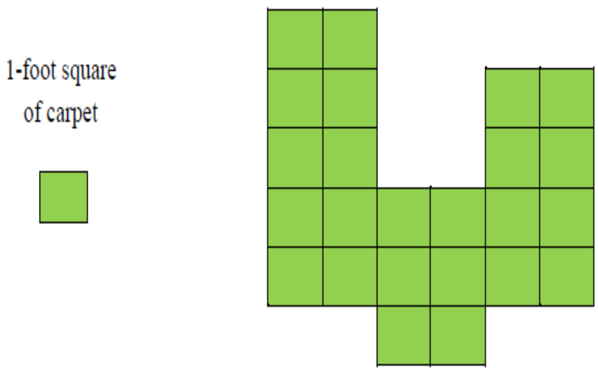
Common Misconceptions
Often students mix up when to 'carry' and when to 'borrow'. Also students often do not notice the need of borrowing and just take the smaller digit from the larger one. Emphasize place value and the meaning of each of the digits.

<u>MGSE4.MD.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems.</u> ***<u>For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.</u>***

Students developed understanding of area and perimeter in 3rd grade by using visual models.

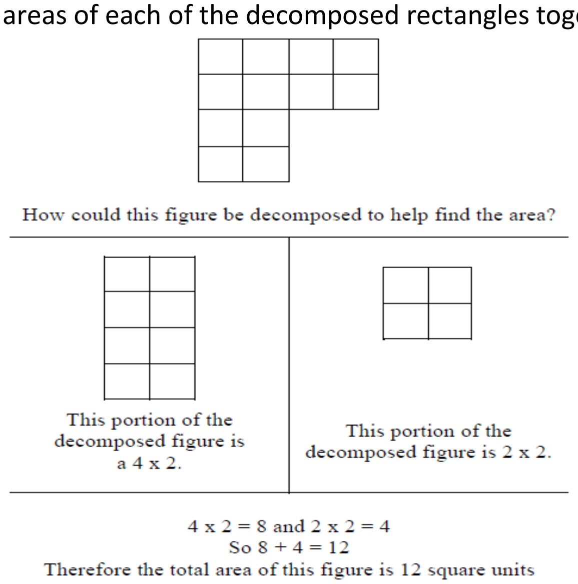
While students are expected to use formulas to calculate area and perimeter of rectangles, they need to understand and be able to communicate their understanding of why the formulas work. The formula for area is l x w and the answer will always be in square units. The formula for perimeter can be 2 l + 2 w or 2 (l + w) and the answer will be in linear units. This standard calls for students to generalize their understanding of area and perimeter by connecting the concepts to mathematical formulas. These formulas should be developed through experience not just memorization.

Example:
Mr. Rutherford is covering the miniature golf course with an artificial grass. How many 1-foot squares of carpet will he need to cover the entire course?

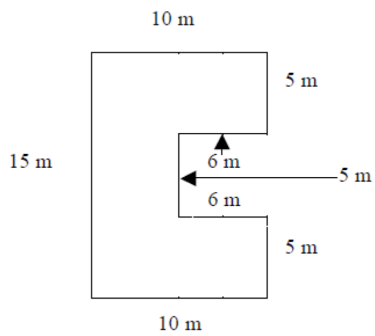


<u>MGSE4.MD.8 Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems</u>.

This standard uses the word rectilinear. A rectilinear figure is a polygon that has all right angles. Students can decompose a rectilinear figure into different rectangles. They find the area of the rectilinear figure by adding the areas of each of the decomposed rectangles together.



Example:
A storage shed is pictured below. What is the total area? How could the figure be decomposed to help find the area?



I can divide this figure into three smaller rectangles.

First: 10 m x 5 m = 50 m²

Second: 5 m x 4 m = 20 m²

Third: 10 m x 5 m = 50 m²

50 m² + 50 m² + 20 m² = 120 m²

(Adapted from Henry County Schools)

<u>Clarification of Standards for Parents</u>
<u>Grade 4 Mathematics Unit 3</u>

Dear Parents,

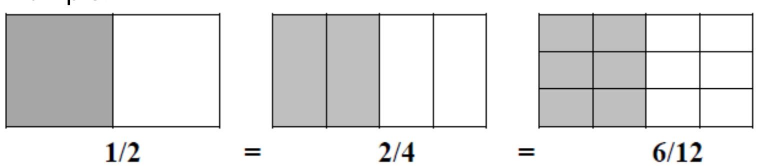
We want to make sure that you have an understanding of the mathematics your child will be learning this year. Below you will find the standards we will be learning in Unit Three. Each standard is in bold print and underlined and below it is an explanation with student examples. Your child is not learning math the way we did when we were in school, so hopefully this will assist you when you help your child at home. Please let your teacher know if you have any questions. ☺

<u>MGSE4.NF.1 Explain why a fraction *a*/*b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.</u>

This standard refers to visual fraction models. This includes area models, number lines or it could be a collection/set model. This standard extends the work in third grade by using additional denominators (5, 10, 12, and 100).

This standard addresses equivalent fractions by examining the idea that equivalent fractions can be created by multiplying both the numerator and denominator by the same number or by dividing a shaded region into various parts.

Example:



Technology Connection: http://illuminations.nctm.org/activitydetail.aspx?id=80

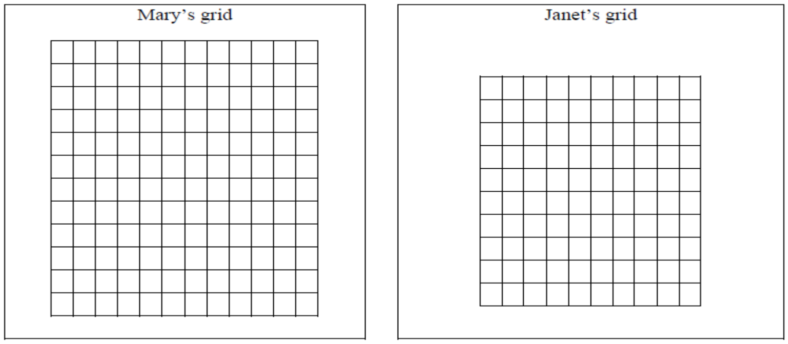
<u>MGSE4.NF.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.</u>

This standard calls students to compare fractions by creating visual fraction models or finding common denominators or numerators. Students' experiences should focus on visual fraction models rather than algorithms. Students should learn to draw fraction models to help them compare. Students must also recognize that they must consider the size of the whole when comparing fractions (i.e., 1/2 and 1/8 of two medium pizzas is very different from 1/2 of one medium and 1/8 of one large).

Example:
Use patterns blocks.

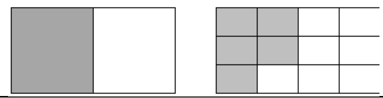
1. If a red trapezoid is one whole, which block shows 1/3?
2. If the blue rhombus is 1/3, which block shows one whole?
3. If the red trapezoid is one whole, which block shows 2/3?

Example:
Mary used a 12 × 12 grid to represent 1 and Janet used a 10 × 10 grid to represent 1. Each girl shaded grid squares to show ¼. How many grid squares did Mary shade? How many grid squares did Janet shade? Why did they need to shade different numbers of grid squares?
Possible solution: Mary shaded 36 grid squares; Janet shaded 25 grid squares. The total number of little squares is different in the two grids, so ¼ of each total number is different.

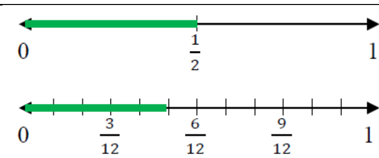


Example:
There are two cakes on the counter that are the same size. The first cake has 1/2 of it left. The second cake has 5/12 left. Which cake has more left?

Student 1: ***Area Model***
The first cake has more left over. The second cake has 5/12 left which is smaller than 1/2.

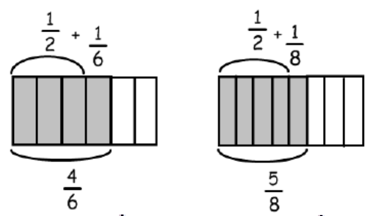


Student 2: ***Number Line Model***
The first cake has more left over: 1/2 is bigger than 5/12.



Student 3: ***Verbal Explanation***
I know that 6/12 equals 1/2, and 5/12 is less than 1/2. Therefore, the second cake has less left over than the first cake. The first cake has more left over.

Example:
When using the benchmark of $\frac{1}{2}$ to compare to $\frac{4}{6}$ and $\frac{5}{8}$, you could use diagrams such as these:



$\frac{4}{6}$ is $\frac{1}{6}$ larger than $\frac{1}{2}$, while $\frac{5}{8}$ is $\frac{1}{8}$ larger than $\frac{1}{2}$. Since $\frac{1}{6}$ is greater than $\frac{1}{8}$, $\frac{4}{6}$ is the greater fraction.

Common Misconceptions

Students think that when generating equivalent fractions they need to multiply or divide either the numerator or denominator, such as, changing 12 to sixths. They would multiply the denominator by 3 to get 16, instead of multiplying the numerator by 3 also. Their focus is only on the multiple of the denominator, not the whole fraction.
Students need to use a fraction in the form of one such as 33 so that the numerator and denominator do not contain the original numerator or denominator.

MGSE4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

The focus in this standard is to have students use and discuss various strategies. It refers to estimation strategies, including using compatible numbers (numbers that sum to 10 or 100) or rounding. Problems should be structured so that all acceptable estimation strategies will arrive at a reasonable answer.

Example 1:
On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many miles did they travel total?

Some typical estimation strategies for this problem are shown below.

Student 1	**Student 2**	**Student 3**
I first thought about 267 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500.	I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundreds that I already had, I end up with 500.	I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200, and 30, I know my answer will be about 530.

The assessment of estimation strategies should only have one reasonable answer (500 or 530), or a range (between 500 and 550). Problems will be structured so that all acceptable estimation strategies will arrive at a reasonable answer.

Example 2:
Your class is collecting bottled water for a service project. The goal is to collect 300 bottles of water. On the first day, Max brings in 3 packs with 6 bottles in each container. Sarah wheels in 6 packs with 6 bottles in each container. About how many bottles of water still need to be collected?

Student 1	**Student 2**
First I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 plus 36 is about 50. I'm trying to get to 300. 50 plus another 50 is 100. Then I need 2 more hundreds. So we still need 250 bottles.	First I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 is about 20 and 36 is about 40. 40 + 20 = 60. 300 – 60 = 240, so we need about 240 more bottles.

This standard references interpreting remainders. Remainders should be put into context for interpretation. Ways to address remainders:

- Remain as a left over
- Partitioned into fractions or decimals
- Discarded leaving only the whole number answer
- Increase the whole number answer up one
- Round to the nearest whole number for an approximate result

Example:
Write different word problems involving **44 ÷ 6 = ?** where the answers are best represented as:

- Problem A: 7
- Problem B: 7 r 2
- Problem C: 8
- Problem D: 7 or 8
- Problem E: $7\ {}^{2}/_{6}$

Possible solutions:

- Problem **A: 7.**

Mary had 44 pencils. Six pencils fit into each of her pencil pouches. How many pouches did she fill? 44 ÷ *6 = p; p = 7 r 2. Mary can fill 7 pouches completely.*

- Problem **B: 7 r 2.**
  Mary had 44 pencils. Six pencils fit into each of her pencil pouches. How many pouches could she fill and how many pencils would she have left? 44 ÷ *6 = p; p = 7 r 2; Mary can fill 7 pouches and have 2 left over.*

- Problem **C: 8.**
  Mary had 44 pencils. Six pencils fit into each of her pencil pouches. What would the fewest number of pouches she would need in order to hold all of her pencils? 44 ÷ *6 = p; p = 7 r 2; Mary can needs 8 pouches to hold all of the pencils.*

- Problem **D: 7 or 8.**
  Mary had 44 pencils. She divided them equally among her friends before giving one of the leftovers to each of her friends. How many pencils could her friends have received? 44 ÷ *6 = p; p = 7 r 2; Some of her friends received 7 pencils. Two friends received 8 pencils.*

- Problem **E: $7^2/_6$.**
  Mary had 44 pencils and put six pencils in each pouch. What fraction represents the number of pouches that Mary filled? 44 ÷ *6 = p; p =* $\mathbf{7}^2/_6$
  Example:
  There are 128 students going on a field trip. If each bus held 30 students, how many buses are needed? *(128 ÷ 30 = b; b = 4 R 8; They will need 5 buses because 4 busses would not hold all of the students).*

Students need to realize in problems, such as the example above, that an extra bus is needed for the 8 students that are left over. Estimation skills include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of situations using various estimation strategies. Estimation strategies include, but are not limited to the following.

- **Front-end estimation with adjusting** (Using the highest place value and estimating from the front end, making adjustments to the estimate by taking into account the remaining amounts)
- **Clustering around an average** (When the values are close together an average value is selected and multiplied by the number of values to determine an estimate.)
- **Rounding and adjusting** (Students round down or round up and then adjust their estimate depending on how much the rounding affected the original values.)
- **Using friendly or compatible numbers such as factors** (Students seek to fit numbers together; e.g., rounding to factors and grouping numbers together that have round sums like 100 or 1000.)
- **Using benchmark numbers that are easy to compute** (Students select close whole numbers for fractions or decimals to determine an estimate.)

(Adapted from Henry County Schools)

<u>Clarification of Standards for Parents</u>

<u>Grade 4 Mathematics Unit 4</u>

Dear Parents,

We want to make sure that you have an understanding of the mathematics your child will be learning this year. Below you will find the standards we will be learning in Unit Four. Each standard is in bold print and underlined and below it is an explanation with student examples. Your child is not learning math the way we did when we were in school, so hopefully this will assist you when you help your child at home. Please let your teacher know if you have any questions. ☺

<u>MGSE4.NF.3 Understand a fraction *a*/*b* with *a* > 1 as a sum of fractions 1/*b*.</u>

a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.

A fraction with a numerator of one is called a unit fraction. When students investigate fractions other than unit fractions, such as 2/3, they should be able to join (compose) or separate (decompose) the fractions of the same whole.

Example: $\frac{2}{3} = \frac{1}{3} + \frac{1}{3}$

Being able to visualize this decomposition into unit fractions helps students when adding or subtracting fractions. Students need multiple opportunities to work with mixed numbers and be able to decompose them in more than one way. Students may use visual models to help develop this understanding.

Example: $1\frac{1}{4} - \frac{3}{4} = ? \quad \rightarrow \quad \frac{4}{4} + \frac{1}{4} = \frac{5}{4} \quad \rightarrow \quad \frac{5}{4} - \frac{3}{4} = \frac{2}{4}\ or\ \frac{1}{2}$

Example of word problem:

Mary and Lacey decide to share a pizza. Mary ate $\frac{3}{6}$ and Lacey ate $\frac{2}{6}$ of the pizza. How much of the pizza did the girls eat together?

Possible solution: The amount of pizza Mary ate can be thought of a $\frac{3}{6}$ or $\frac{1}{6} + \frac{1}{6} + \frac{1}{6}$. The amount of pizza Lacey ate can be thought of a $\frac{1}{6} + \frac{1}{6}$. The total amount of pizza they ate is $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$ or $\frac{5}{6}$ of the pizza. A fraction with a numerator of one is called a unit fraction. When students investigate fractions other than unit fractions, such as 2/3, they should be able to join (compose) or separate (decompose) the fractions of the same whole.

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Example of word problem:
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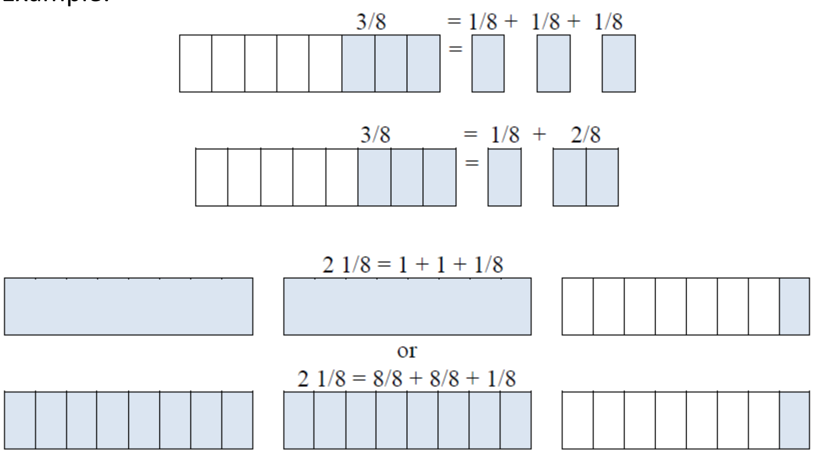
b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*

Students should justify their breaking apart (decomposing) of fractions using visual fraction models. The concept of turning mixed numbers into improper fractions needs to be emphasized using visual fraction models.

Example:



c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.

A separate algorithm for mixed numbers in addition and subtraction is not necessary. Students will tend to add or subtract the whole numbers first and then work with the fractions using the same strategies they have applied to problems that contained only fractions.

Example:

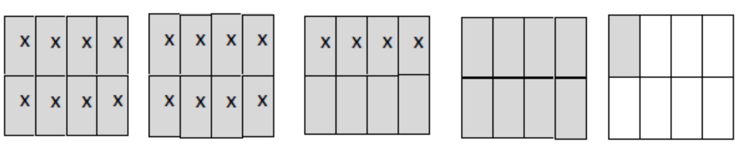
Susan and Maria need $8\frac{3}{8}$ feet of ribbon to package gift baskets. Susan has $3\frac{1}{8}$ feet of ribbon and Maria has $5\frac{3}{8}$ feet of ribbon. How much ribbon do they have altogether? Will it be enough to complete the project? Explain why or why not.

The student thinks: I can add the ribbon Susan has to the ribbon Maria has to find out how much ribbon they have altogether. Susan has $3\frac{1}{8}$ feet of ribbon and Maria has $5\frac{3}{8}$ feet of ribbon. I can write this as $3\frac{1}{8} + 5\frac{3}{8}$. I know they have 8 feet of ribbon by adding the 3 and 5. They also have $\frac{1}{8}$ and $\frac{3}{8}$ which makes a total of $\frac{4}{8}$ more. Altogether they have $8\frac{4}{8}$ feet of ribbon. $8\frac{4}{8}$8 is larger than $8\frac{3}{8}$ so they will have enough ribbon to complete the project. They will even have a little extra ribbon left: $\frac{1}{8}$ foot.

Example:

Trevor has $4\frac{1}{8}$ pizzas left over from his soccer party. After giving some pizza to his friend, he has $2\frac{4}{8}$ of a pizza left. How much pizza did Trevor give to his friend?

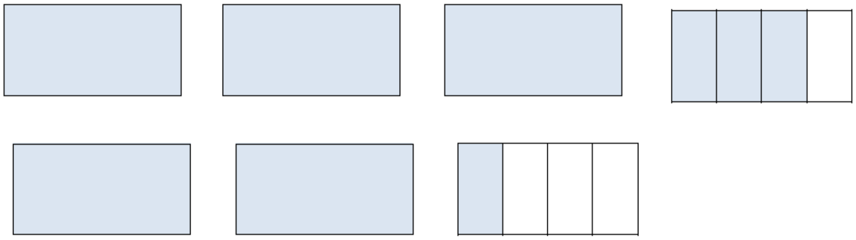
Possible solution: Trevor had $4\frac{1}{8}$ pizzas to start. This is $\frac{33}{8}$ of a pizza. The x's show the pizza he has left which is $2\frac{4}{8}$ pizzas or $\frac{20}{8}$ pizzas. The shaded rectangles without the x's are the pizza he gave to his friend which is $\frac{13}{8}$ or $1\frac{5}{8}$ pizzas.



Mixed numbers are introduced for the first time in 4th Grade. Students should have ample experiences of adding and subtracting mixed numbers where they work with mixed numbers or convert mixed numbers into improper fractions.

Example:

While solving the problem, $3\frac{3}{4} + 2\frac{1}{4}$, students could do the following:



Student 1: $3 + 2 = 5$ and $\frac{3}{4} + \frac{1}{4} = 1$, so $5 + 1 = 6$.

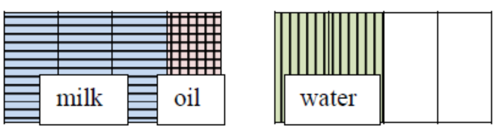
Student 2: $3\frac{3}{4} + 2 = 5\frac{3}{4}$, so $5\frac{3}{4} + \frac{1}{4} = 6$.

Student 3: $3\frac{3}{4} = \frac{15}{4}$ and $2\frac{1}{4} = \frac{9}{4}$, so $\frac{15}{4} + \frac{9}{4} = \frac{24}{4} = 6$.

d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

Example:

A cake recipe calls for you to use $\frac{3}{4}$ cup of milk, $\frac{1}{4}$ cup of oil, and $\frac{2}{4}$ cup of water. How much liquid was needed to make the cake?



3/4 + 1/4 + 2/4 = 6/4 = 1 2/4

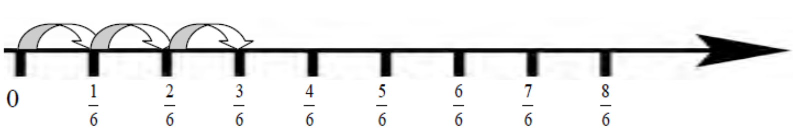
<u>MGSE4.NF.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.</u>

a. Understand a fraction *a*/*b* as a multiple of 1/*b*. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*

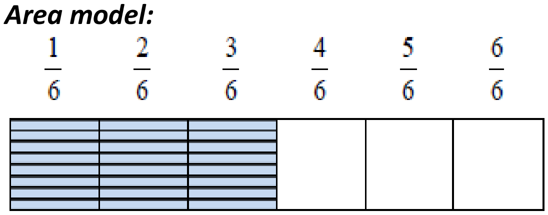
This standard builds on students' work of adding fractions and extending that work into multiplication.

Example: $\frac{3}{6} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 3 \times \frac{1}{6}$

Number line:



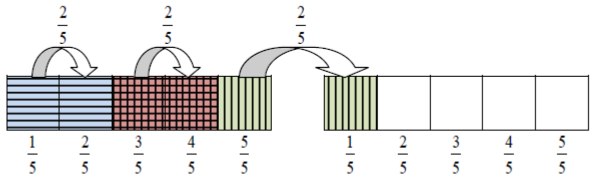
Area model:



b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. ***For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)***

This standard extended the idea of multiplication as repeated addition. For example, $3 \times \frac{2}{5} = \frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \frac{6}{5} = 6 \times \frac{1}{5}$.

Students are expected to use and create visual fraction models to multiply a whole number by a fraction.



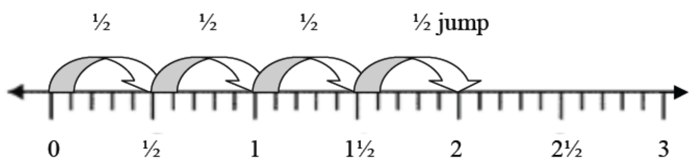
c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. ***For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?***

This standard calls for students to use visual fraction models to solve word problems related to multiplying a whole number by a fraction.

Example:

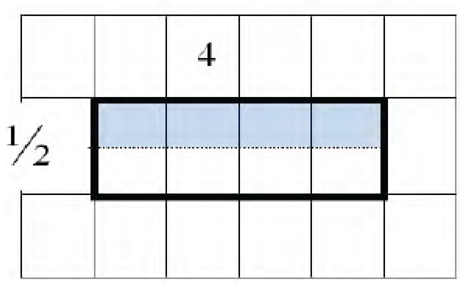
In a relay race, each runner runs ½ of a lap. If there are 4 team members how long is the race?

Student 1 – Draws a number line showing 4 jumps of ½:



Student 2 – Draws an area model showing 4 pieces of ½ joined together to equal 2:

Student 3 – Draws an area model representing 4 × ½ on a grid, dividing one row into ½ to represent the multiplier:



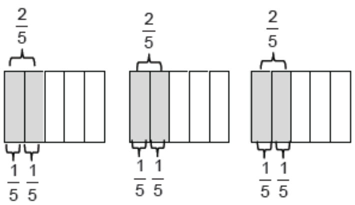
Example:

Heather bought 12 plums and ate 13 of them. Paul bought 12 plums and ate 14 of them. Which statement is true? Draw a model to explain your reasoning.

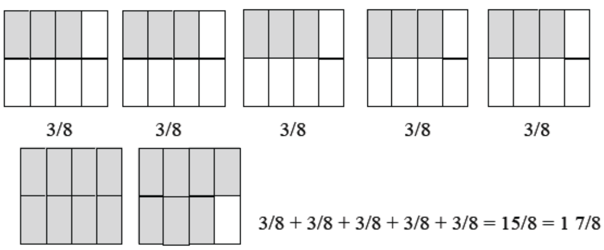
a. Heather and Paul ate the same number of plums.
b. Heather ate 4 plums and Paul ate 3 plums.
c. Heather ate 3 plums and Paul ate 4 plums.
d. Heather had 9 plums remaining.

Examples:
Students need many opportunities to work with problems in context to understand the connections between models and corresponding equations. Contexts involving a whole number times a fraction lend themselves to modeling and examining patterns.

1. $3 \times \frac{2}{5} = 6 \times \frac{1}{5} = \frac{6}{5}$



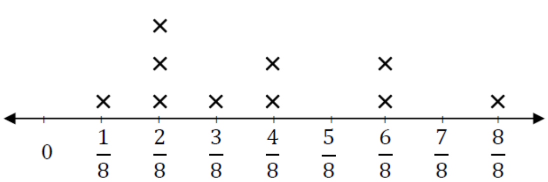
2. If each person at a party eats 38 of a pound of roast beef, and there are 5 people at the party, how many pounds of roast beef are needed? Between what two whole numbers does your answer lie?
A student may build a fraction model to represent this problem:



<u>Common Misconceptions</u>
Students think that it does not matter which model to use when finding the sum or difference of fractions. They may represent one fraction with a rectangle and the other fraction with a circle. They need to know that the models need to represent the same whole.

<u>MGSE4.MD.4 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*</u>

This standard provides a context for students to work with fractions by measuring objects to an eighth of an inch. Students are making a line plot of this data and then adding and subtracting fractions based on data in the line plot.
Example:
Students measured objects in their desk to the nearest 1/2, 1/4, or 1/8 inch. They displayed their data collected on a line plot. How many objects measured 1/4 inch? 1/2 inch? If you put all the objects together end to end what would be the total length of **all** the objects.



<u>Common Misconceptions</u>
Students use whole-number names when counting fractional parts on a number line. The fraction name should be used instead. For example, if two-fourths is represented on the line plot three times, then there would be six-fourths.

<u>MGSE4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.</u>
The focus in this standard is to have students use and discuss various strategies. It refers to estimation strategies, including using compatible numbers (numbers that sum to 10 or 100) or rounding. Problems should be structured so that all acceptable estimation strategies will arrive at a reasonable answer.

Example 1:
On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many miles did they travel total?

Some typical estimation strategies for this problem are shown below.

Student 1	**Student 2**	**Student 3**
I first thought about 267 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500.	I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundreds that I already had, I end up with 500.	I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200, and 30, I know my answer will be about 530.

The assessment of estimation strategies should only have one reasonable answer (500 or 530), or a range (between 500 and 550). Problems will be structured so that all acceptable estimation strategies will arrive at a reasonable answer.

Example 2:
Your class is collecting bottled water for a service project. The goal is to collect 300 bottles of water. On the first day, Max brings in 3 packs with 6 bottles in each container. Sarah wheels in 6 packs with 6 bottles in each container. About how many bottles of water still need to be collected?

Student 1	**Student 2**
First I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 plus 36 is about 50. I'm trying to get to 300. 50 plus another 50 is 100. Then I need 2 more hundreds. So we still need 250 bottles.	First I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 is about 20 and 36 is about 40. 40 + 20 = 60. 300 – 60 = 240, so we need about 240 more bottles.

This standard references interpreting remainders. Remainders should be put into context for interpretation. Ways to address remainders:

- Remain as a left over
- Partitioned into fractions or decimals
- Discarded leaving only the whole number answer
- Increase the whole number answer up one
- Round to the nearest whole number for an approximate result

Example:
Write different word problems involving **44 ÷ 6 = ?** where the answers are best represented as:

- Problem A: 7

- Problem B: 7 r 2
- Problem C: 8
- Problem D: 7 or 8
- Problem E: 7 $^2/_6$

Possible solutions:

- Problem **A: 7.**
  Mary had 44 pencils. Six pencils fit into each of her pencil pouches. How many pouches did she fill? 44 ÷ *6 = p; p = 7 r 2. Mary can fill 7 pouches completely.*

- Problem **B: 7 r 2.**
  Mary had 44 pencils. Six pencils fit into each of her pencil pouches. How many pouches could she fill and how many pencils would she have left? 44 ÷ *6 = p; p = 7 r 2; Mary can fill 7 pouches and have 2 left over.*

- Problem **C: 8.**
  Mary had 44 pencils. Six pencils fit into each of her pencil pouches. What would the fewest number of pouches she would need in order to hold all of her pencils? 44 ÷ *6 = p; p = 7 r 2; Mary can needs 8 pouches to hold all of the pencils.*

- Problem **D: 7 or 8.**
  Mary had 44 pencils. She divided them equally among her friends before giving one of the leftovers to each of her friends. How many pencils could her friends have received? 44 ÷ *6 = p; p = 7 r 2; some of her friends received 7 pencils. Two friends received 8 pencils.*

- Problem **E: $7^2/_6$.**
  Mary had 44 pencils and put six pencils in each pouch. What fraction represents the number of pouches that Mary filled? 44 ÷ *6 = p; p =* $\mathbf{7}^2/_6$
  Example:
  There are 128 students going on a field trip. If each bus held 30 students, how many buses are needed? *(128 ÷ 30 = b; b = 4 R 8; they will need 5 buses because 4 busses would not hold all of the students).*

Students need to realize in problems, such as the example above, that an extra bus is needed for the 8 students that are left over. Estimation skills include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of situations using various estimation strategies. Estimation strategies include, but are not limited to the following.

- **Front-end estimation with adjusting** (Using the highest place value and estimating from the front end, making adjustments to the estimate by taking into account the remaining amounts)
- **Clustering around an average** (When the values are close together an average value is selected and multiplied by the number of values to determine an estimate.)
- **Rounding and adjusting** (Students round down or round up and then adjust their estimate depending on how much the rounding affected the original values.)
- **Using friendly or compatible numbers such as factors** (Students seek to fit numbers together; e.g., rounding to factors and grouping numbers together that have round sums like 100 or 1000.)
- **Using benchmark numbers that are easy to compute** (Students select close whole numbers for fractions or decimals to determine an estimate.)

(Adapted from Henry County Schools)

Clarification of Standards for Parents

Grade 4 Mathematics Unit 5

Dear Parents,

We want to make sure that you have an understanding of the mathematics your child will be learning this year. Below you will find the standards we will be learning in Unit Four. Each standard is in bold print and underlined and below it is an explanation with student examples. Your child is not learning math the way we did when we were in school, so hopefully this will assist you when you help your child at home. Please let your teacher know if you have any questions. ☺

MGSE4.NF.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.*

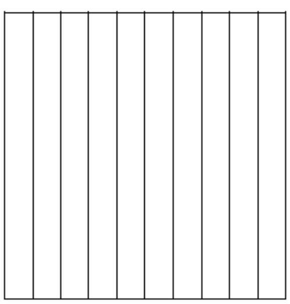
This standard continues the work of equivalent fractions by having students change fractions with a 10 in the denominator into equivalent fractions that have a 100 in the denominator. In order to prepare for work with decimals (MGSE4.NF.6 and MGSE4.NF.7), experiences that allow students to shade decimal grids (10×10 grids) can support this work. Student experiences should focus on working with grids rather than algorithms. Students can also use base ten blocks and other place value models to explore the relationship between fractions with denominators of 10 and denominators of 100.

This work in 4^{th} grade lays the foundation for performing operations with decimal numbers in 5^{th} grade.

Example:

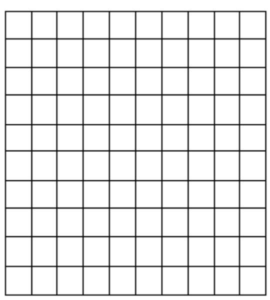
Ones	.	Tenths	Hundredths

Tenths Grid



.3 = 3 tenths = 3/10

Hundredths Grid

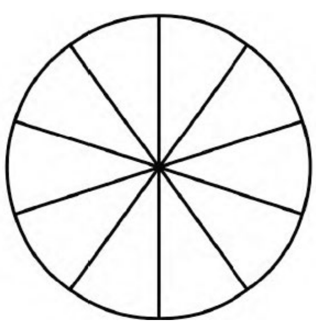


.30 = 30 hundredths = 30/100

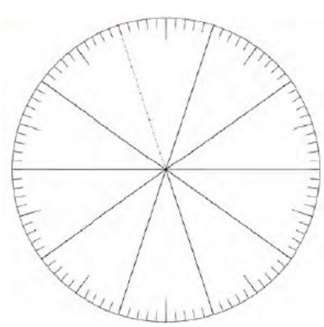
Example:

Represent 3 tenths and 30 hundredths on the models below.

Tenths circle



Hundredths circle



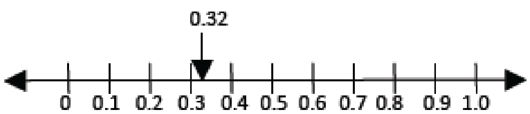
MGSE4.NF.6 Use decimal notation for fractions with denominators 10 or 100. ***For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.***

Decimals are introduced for the first time. Students should have ample opportunities to explore and reason about the idea that a number can be represented as both a fraction and a decimal.

Students make connections between fractions with denominators of 10 and 100 and the place value chart. By reading fraction names, students say $\frac{32}{100}$ as thirty-two hundredths and rewrite this as 0.32 or represent it on a place value model as shown below.

Hundreds	Tens	Ones	•	Tenths	Hundredths
			•	3	2

Students use the representations explored in MCC.4.NF.5 to understand $\frac{32}{100}$ can be expanded to $\frac{3}{10}$ and $\frac{2}{100}$. Students represent values such as 0.32 or $\frac{32}{100}$ on a number line. $\frac{32}{100}$ is more than $\frac{30}{100}$ (or $\frac{3}{10}$) and less than $\frac{40}{100}$ (or $\frac{4}{10}$). It is closer to $\frac{30}{100}$ so it would be placed on the number line near that value.



MGSE4.NF.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

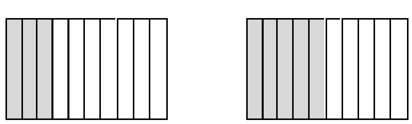
Students should reason that comparisons are only valid when they refer to the same whole. Visual models include area models, decimal grids, decimal circles, number lines, and meter sticks.

Students build area and other models to compare decimals. Through these experiences and their work with fraction models, they build the understanding that comparisons between decimals or fractions are only valid when the whole is the same for both cases. Each of the models below shows 3/10 but the whole on the right is much bigger than the whole on the left. They are both 3/10 but the model on the right is a much larger quantity than the model on the left.

When the wholes are the same, the decimals or fractions can be compared.

Example:

Draw a model to show that 0.3 < 0.5. (*Students would sketch two models of approximately the same size to show the area that represents three-tenths is smaller than the area that represents five-tenths.*)



Common Misconceptions

Students treat decimals as whole numbers when making comparison of two decimals. They think the longer the number, the greater the value. For example, they think that 0.03 is greater than 0.3.

MGSE4.OA.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

The focus in this standard is to have students use and discuss various strategies. It refers to estimation strategies, including using compatible numbers (numbers that sum to 10 or 100) or rounding. Problems should be structured so that all acceptable estimation strategies will arrive at a reasonable answer.

Example 1:
On a vacation, your family travels 267 miles on the first day, 194 miles on the second day and 34 miles on the third day. How many miles did they travel total?

Some typical estimation strategies for this problem are shown below.

Student 1	**Student 2**	**Student 3**
I first thought about 267 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500.	I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundreds that I already had, I end up with 500.	I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200, and 30, I know my answer will be about 530.

The assessment of estimation strategies should only have one reasonable answer (500 or 530), or a range (between 500 and 550). Problems will be structured so that all acceptable estimation strategies will arrive at a reasonable answer.

Example 2:
Your class is collecting bottled water for a service project. The goal is to collect 300 bottles of water. On the first day, Max brings in 3 packs with 6 bottles in each container. Sarah wheels in 6 packs with 6 bottles in each container. About how many bottles of water still need to be collected?

Student 1	**Student 2**
First I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 plus 36 is about 50. I'm trying to get to 300. 50 plus another 50 is 100. Then I need 2 more hundreds. So we still need 250 bottles.	First I multiplied 3 and 6 which equals 18. Then I multiplied 6 and 6 which is 36. I know 18 is about 20 and 36 is about 40. 40 + 20 = 60. 300 – 60 = 240, so we need about 240 more bottles.

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Example:
Write different word problems involving **44 ÷ 6 = ?** where the answers are best represented as:

- Problem A: 7
- Problem B: 7 r 2
- Problem C: 8
- Problem D: 7 or 8
- Problem E: $7\ ^{2}/_{6}$

Possible solutions:

- Problem **A: 7.**
  Mary had 44 pencils. Six pencils fit into each of her pencil pouches. How many pouches did she fill? 44 ÷ *6 = p; p = 7 r 2. Mary can fill 7 pouches completely.*

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- Problem **C: 8.**
  Mary had 44 pencils. Six pencils fit into each of her pencil pouches. What would the fewest number of pouches she would need in order to hold all of her pencils? 44 ÷ *6 = p; p = 7 r 2; Mary can needs 8 pouches to hold all of the pencils.*

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  Mary had 44 pencils. She divided them equally among her friends before giving one of the leftovers to each of her friends. How many pencils could her friends have received? 44 ÷ *6 = p; p = 7 r 2; some of her friends received 7 pencils. Two friends received 8 pencils.*

- Problem **E: $7^2/_6$.**
  Mary had 44 pencils and put six pencils in each pouch. What fraction represents the number of pouches that Mary filled? 44 ÷ *6 = p; p =* $\mathbf{7}^2/_6$
  Example:
  There are 128 students going on a field trip. If each bus held 30 students, how many buses are needed? *(128 ÷ 30 = b; b = 4 R 8; they will need 5 buses because 4 busses would not hold all of the students).*

Students need to realize in problems, such as the example above, that an extra bus is needed for the 8 students that are left over. Estimation skills include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of situations using various estimation strategies. Estimation strategies include, but are not limited to the following.

- **Front-end estimation with adjusting** (Using the highest place value and estimating from the front end, making adjustments to the estimate by taking into account the remaining amounts)
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- **Using friendly or compatible numbers such as factors** (Students seek to fit numbers together; e.g., rounding to factors and grouping numbers together that have round sums like 100 or 1000.)
- **Using benchmark numbers that are easy to compute** (Students select close whole numbers for fractions or decimals to determine an estimate.)

MGSE4.MD.2 Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.
This standard includes multi-step word problems related to expressing measurements from a larger unit in terms of a smaller unit (e.g., feet to inches, meters to centimeter, dollars to cents). Students should have ample opportunities to use number line diagrams to solve word problems.

In unit one, students focus on solving measurement word problems that involve the operations of addition and subtraction. Students also use diagrams (such as number line diagrams) to solve problems.

Example:

Mason ran for an hour and 15 minutes on Monday, 25 minutes on Tuesday, and 40 minutes on Wednesday. What was the total number of minutes Mason ran?

Students can add the times to find the total number of minutes Mason ran. 40 minutes plus another 25 minutes would be 65 minutes, or an hour and 5 minutes. Then, an hour and five minutes can be added to an hour and 15 minutes to see that Mason ran 2 hours and 20 minutes in all.

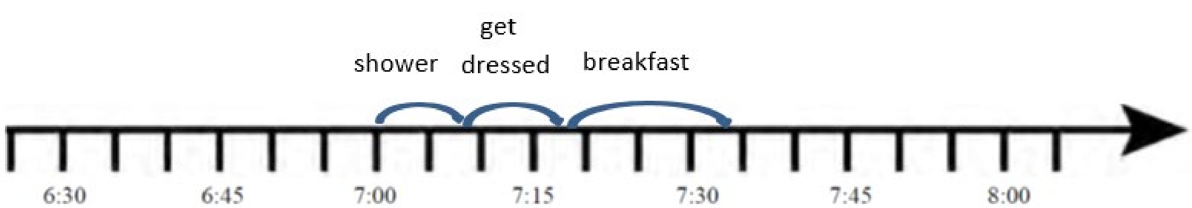
Example:
A pound of apples costs $1.20. Rachel bought a pound and a half of apples. If she gave the clerk a $5.00 bill, how much change will she get back?

Possible student solution: If Rachel bought a pound and a half of apples, she paid $1.20 for the first pound and then 60¢ for the other half a pound, since half of $1.20 is 60¢. When I add $1.20 and 60¢, I get a total of $1.80 spent on the apples. If she gave the clerk a five dollar bill, I can count up to find out how much change she received.

> $1.80 + 20¢ = $2.00
> $2.00 + 3.00 = $5.00
> $3.00 + 20¢ = $3.20
> So Rachel got $3.20 back in change.

Number line diagrams that feature a measurement scale can represent measurement quantities. Examples include: ruler, diagram marking off distance along a road with cities at various points, a timetable showing hours throughout the day, or a liquid volume measure on the side of a container.

Example:
At 7:00 a.m. Candace wakes up to go to school. It takes her 8 minutes to shower, 9 minutes to get dressed and 17 minutes to eat breakfast. How many minutes does she have until the bus comes at 8:00 a.m.? Use the number line to help solve the problem.



Candace is finished at 7:34. If the bus comes at 8:00, I can count on to from 7:34 to 8:00 to find how many minutes it takes for the bus to arrive. From 7:34 to 7:35 is one minute. From 7:35 to 7:40 is 5 minutes and from 7:40 to 8:00 is 20 minutes. 1 minute + 5 minutes + 20 minutes = 26 minutes until the bus arrives.

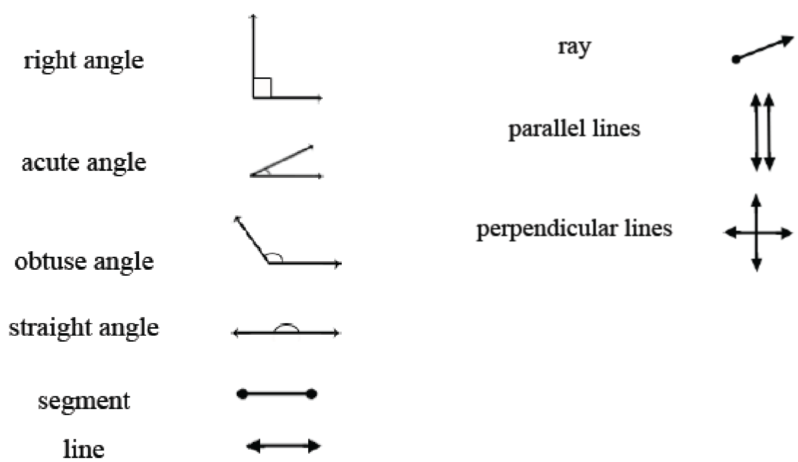
(Adapted from Henry County Schools)

Clarification of Standards for Parents
Grade 4 Mathematics Unit 6

Dear Parents,
We want to make sure that you have an understanding of the mathematics your child will be learning this year. Below you will find the standards we will be learning in Unit Six. Each standard is in bold print and underlined and below it is an explanation with student examples. Your child is not learning math the way we did when we were in school, so hopefully this will assist you when you help your child at home. Please let your teacher know if you have any questions. ☺

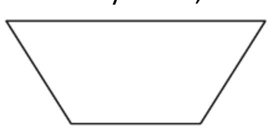
<u>MGSE.4.G.1 Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.</u>

This standard asks students to draw two-dimensional geometric objects and to also identify them in two-dimensional figures. This is the first time that students are exposed to rays, angles, and perpendicular and parallel lines. Examples of points, line segments, lines, angles, parallelism, and perpendicularity can be seen daily. Students do not easily identify lines and rays because they are more abstract.



Example:
Draw two different types of quadrilaterals that have two pairs of parallel sides?
Is it possible to have an acute right triangle? Justify your reasoning using pictures and words.

Examples:
How many acute, obtuse and right angles are in this shape?



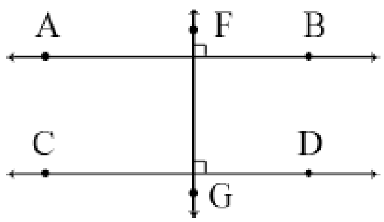
Draw and list the properties of a parallelogram. Draw and list the properties of a rectangle. How are your drawings and lists alike? How are they different? Be ready to share your thinking with the class.

<u>MGSE4.G.2 Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.</u>
Two-dimensional figures may be classified using different characteristics such as, parallel or perpendicular lines or by angle measurement.

Parallel or Perpendicular Lines:
Students should become familiar with the concept of parallel and perpendicular lines. Two lines are parallel if they never intersect and are always equidistant. Two lines are perpendicular if they intersect in right angles (90º). Students may use transparencies with lines to arrange two lines in different ways to determine that the 2 lines might intersect in one point

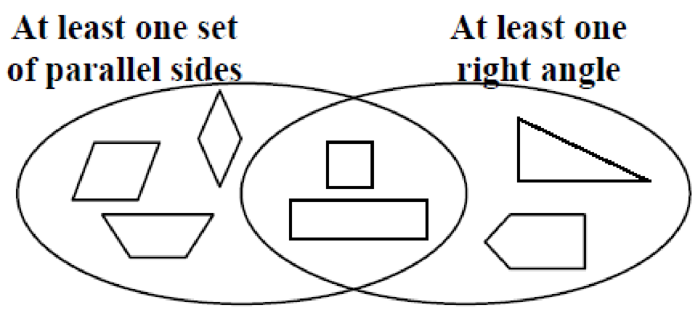
or may never intersect. Further investigations may be initiated using geometry software. These types of explorations may lead to a discussion on angles.

Parallel and perpendicular lines are shown below:



This standard calls for students to sort objects based on parallelism, perpendicularity and angle types.

Example:



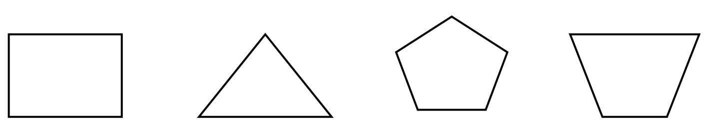
Do you agree with the label on each of the circles in the Venn diagram above? Describe why some shapes fall in the overlapping sections of the circles.

Example:
Draw and name a figure that has two parallel sides and exactly 2 right angles.

Example:
For each of the following, sketch an example if it is possible. If it is impossible, say so, and explain why or show a counterexample.

- A parallelogram with exactly one right angle.
- An isosceles right triangle.
- A rectangle that is *not* a parallelogram. *(impossible)*
- Every square is a quadrilateral.
- Every trapezoid is a parallelogram.

Example:
Identify which of these shapes have perpendicular or parallel sides and justify your selection.



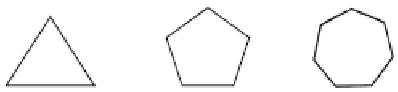
A possible justification that students might give is: "The square has perpendicular lines because the sides meet at a corner, forming right angles."

Angle Measurement:
Students' experiences with drawing and identifying right, acute, and obtuse angles support them in classifying two-dimensional figures based on specified angle measurements. They use the benchmark angles of 90°, 180°, and 360° to approximate the measurement of angles.

Right triangles can be a category for classification. A right triangle has one right angle. There are different types of right triangles. An isosceles right triangle has two or more congruent sides and a scalene right triangle has no congruent sides.

<u>MGSE4.G.3 Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.</u>
Students need experiences with figures which are symmetrical and non-symmetrical. Figures include both regular and non-regular polygons. Folding cut-out figures will help students determine whether a figure has one or more lines of symmetry. This standard only includes line symmetry, not rotational symmetry.

Example:
For each figure at the right, draw all of the lines of symmetry. What pattern do you notice? How many lines of symmetry do you think there would be for regular polygons with 9 and 11 sides. Sketch each figure and check your predictions.



Polygons with an odd number of sides have lines of symmetry that go from a midpoint of a side through a vertex.

<u>Common Misconceptions</u>
Students believe a wide angle with short sides may seem smaller than a narrow angle with long sides. Students can compare two angles by tracing one and placing it over the other. Students will then realize that the length of the sides does not determine whether one angle is larger or smaller than another angle. The measure of the angle does not change.

(Adapted from Henry County Schools)

<u>Clarification of Standards for Parents</u>
<u>Grade 4 Mathematics Unit 7</u>

Dear Parents,
We want to make sure that you have an understanding of the mathematics your child will be learning this year. Below you will find the standards we will be learning in Unit Seven. Each standard is in bold print and underlined and below it is an explanation with student examples. Your child is not learning math the way we did when we were in school, so hopefully this will assist you when you help your child at home. Please let your teacher know if you have any questions. ☺

<u>MGSE4.MD.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table.</u> ***For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...***

The units of measure that have not been addressed in prior years are cups, pints, quarts, gallons, pounds, ounces, kilometers, milliliters, and seconds. Students' prior experiences were limited to measuring length, mass (metric and customary systems), liquid volume (metric only), and elapsed time. Students did not convert measurements. Students need ample opportunities to become familiar with these new units of measure and explore the patterns and relationships in the conversion tables that they create.

Students may use a two-column chart to convert from larger to smaller units and record equivalent measurements. They make statements such as, if one foot is 12 inches, then 3 feet has to be 36 inches because there are 3 groups of 12.

Example:
Customary length conversion table

Yards	Feet
1	3
2	6
3	9
n	$n \times 3$

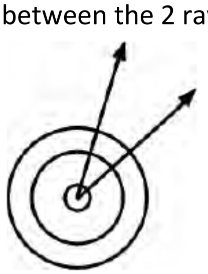
Foundational understandings to help with measure concepts:
- Understand that larger units can be subdivided into equivalent units (partition).
- Understand that the same unit can be repeated to determine the measure (iteration).
- Understand the relationship between the size of a unit and the number of units needed (compensatory principle[1]).

<u>MGSE4.MD.5 Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:</u>

This standard brings up a connection between angles and circular measurement (360 degrees).

a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.

The diagram below will help students understand that an angle measurement is not related to an area since the area between the 2 rays is different for both circles yet the angle measure is the same.

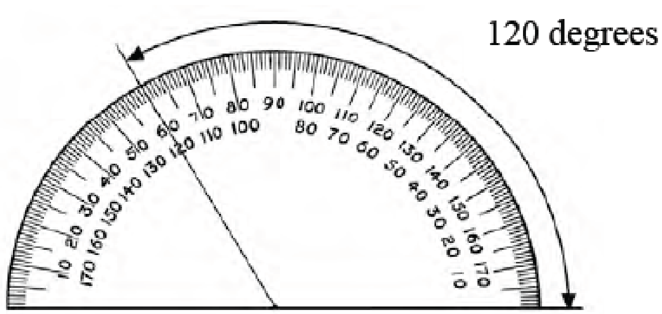


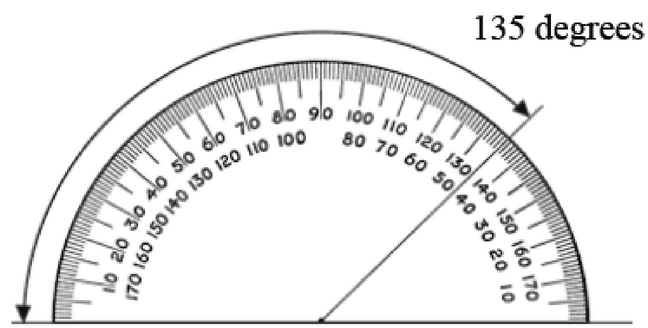
b. An angle that turns through n one-degree angles is said to have an angle measure of n degrees.

This standard calls for students to explore an angle as a series of "one-degree turns." A water sprinkler rotates one-degree at each interval. If the sprinkler rotates a total of 100 degrees, how many one-degree turns has the sprinkler made?

<u>MGSE4.MD.6 Measure angles in whole number degrees using a protractor. Sketch angles of specified measure.</u>
Before students begin measuring angles with protractors, they need to have some experiences with benchmark angles. They transfer their understanding that a 360º rotation about a point makes a complete circle to recognize and sketch angles that measure approximately 90º and 180º. They extend this understanding and recognize and sketch angles that measure approximately 45º and 30º. They use appropriate terminology (acute, right, and obtuse) to describe angles and rays (perpendicular).

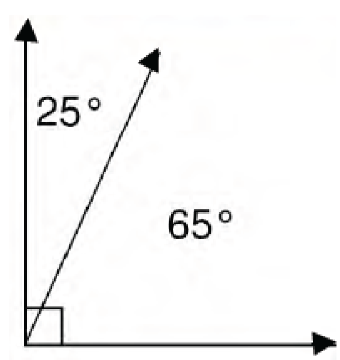
Students should measure angles and sketch angles.





<u>MGSE4.MD.7 Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.</u>

This standard addresses the idea of decomposing (breaking apart) an angle into smaller parts.



Example:

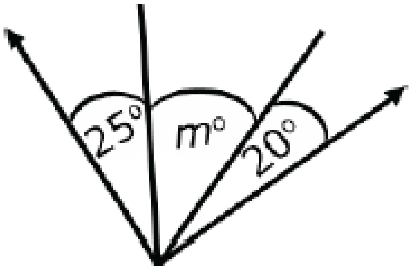
A lawn water sprinkler rotates 65 degrees and then pauses. It then rotates an additional 25 degrees. What is the total degree of the water sprinkler rotation? To cover a full 360 degrees how many times will the water sprinkler need to be moved?

If the water sprinkler rotates a total of 25 degrees then pauses, how many 25 degree cycles will it go through for the rotation to reach at least 90 degrees?

Example:

If the two rays are perpendicular, what is the value of *m*?

Example:



Joey knows that when a clock's hands are exactly on 12 and 1, the angle formed by the clock's hands measures 30º. What is the measure of the angle formed when a clock's hands are exactly on the 12 and 4?

<u>Common Misconceptions</u>

Students are confused as to which number to use when determining the measure of an angle using a protractor because most protractors have a double set of numbers. Students should decide first if the angle appears to be an angle that is less than the measure of a right angle (90°) or greater than the measure of a right angle (90°). If the angle appears to be less than 90°, it is an acute angle and its measure ranges from 0° to 89°. If the angle appears to be an angle that is greater than 90°, it is an obtuse angle and its measures range from 91° to 179°. Ask questions about the appearance of the angle to help students in deciding which number to use.

(Adapted from Henry County Schools)