### **Cobb County School District**



Algebra I Teaching & Learning Framework							
	Seme	ster 1	Semester 2				
Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6		
4 weeks	5 weeks	9 weeks	6 weeks	5 weeks	7 weeks		
Relationships	Reasoning with	Modeling & Analyzing Quadratic	Modeling & Analyzing	Comparing &	Describing Data		
Between	Linear Equations &	Functions	<b>Exponential Functions</b>	Contrasting			
Quantities & Inequalities			Functions	Review & Extend			
Expressions	Expressions						
MGSE9-12.N.RN.2-3	MGSE9-12.A.CED.1-4	MGSE9-12.A.SSE.2	MGSE9-12.A.CED.1-2	MGSE9-12.F.LE.1	MGSE9-12.S.ID.1		
(Properties of rational &	(Create equations that	(Interpret the structure of expressions)	(Create equations 1-2	(Linear vs exponential)	(Dot plots, histograms & box plots)		
irrational numbers)	describe numbers or	MGSE9-12.A.SSE.3,3a-b	variables)	MGSE9-12.F.LE.1a	MGSE9-12.S.ID.2		
MGSE9-12.N.Q.1-3	relationships)	(Equivalent forms of expressions)	MGSE9-12.A.REI.1	(Growth of functions)	(Compare data distribution)		
(Reason quantitatively &	MGSE9-12.A.REI.1,3,5	MGSE9-12.A.CED.1-2,4	(Justify how to solve an	MGSE9-12.F.LE.1b,c,2-3	MGSE9-12.S.ID.3		
use units to solve	(Solve equations &	(Create equations that describe numbers or	equation)	(Changes in rate and	(Shape, center & spread)		
problems)	inequalities 1-2 variable)	relationships)	MGSE9-12.F.BF.1	relating to context)	MGSE9-12.S.ID.5-6		
MGSE9-12.A.SSE.1	MGSE9-12.A.REI.6	MGSE9-12.A.REI.1	(Write a function)	MGSE9-12.F.LE.5	(Bivariate data)		
(Interpret expressions in	(Solve systems)	(Solve systems) (Justify how to solve an equation) MGSE9-12.F.BF.1a	MGSE9-12.F.BF.1a,2	(Interpret parameters)	MGSE9-12.S.ID.6a,c		
context)	MGSE9-12.A.REI.10-12	MGSE9-12.A.REI.4,4a-b	(Arithmetic & geometric	MGSE9-12.F.BF.3	(Function of best fit)		
MGSE9-12.A.SSE.1a-b	(Solve equations &	(Methods of solving quadratics)	sequences)	(Build new functions)	MGSE9-12.S.ID.7-9		
(Interpret formulas &	inequalities 2 variables)	MGSE9-12.F.BF.1,3	MGSE9-12.F.BF.3	MGSE9-12.F.IF.1	(Slope, correlation coefficient,		
expressions in context)	expressions in context) MGSE9-12.F.BF.1 (Write a function & build new functions)		(Build new functions)	(Input vs. output)	causation & correlation)		
MGSE9-12.A.APR.1	MGSE9-12.A.APR.1 (Write a function) MGSE9-12.F.IF.1		MGSE9-12.F.IF.1	MGSE9-12.F.IF.2			
(Add, subtract & multiply	MGSE9-12.F.BF.1a,2	(Input vs. output)	(Input vs. output)	(Function notation)			
polynomials) (Arithmetic & geometric		MGSE9-12.F.IF.2	MGSE9-12.F.IF.2	MGSE9-12.F.IF.4	Review: All standards by		
	sequences)	(Function notation)	(Function notation)	(Characteristics)	differentiating for student needs		
	MGSE9-12.F.IF.1	MGSE9-12.F.IF.4	MGSE9-12.F.IF.3-4	MGSE9-12.F.IF.5-6			
	(Input vs. output)	(Characteristics)	(Sequences &	(Rate of change)	Extend:		
	MGSE9-12.F.IF.2	MGSE9-12.F.IF.5-6	characteristics)	MGSE9-12.F.IF.7	MGSE9-12 G CO 1		
	(Function notation)	(Rate of change)	MGSE9-12.F.IF.5-6	(Graph functions)	(Precise definitions)		
	MGSE9-12.F.IF.3-4	MGSE9-12.F.IF.7,7a	(Rate of change)	MGSE9-12.F.IF.9			
	(Sequences & characteristics)	(Graph functions)	MGSE9-12.F.IF.7,7e	(Compare functions)			
	MGSE9-12.F.IF.5-6	MGSE9-12.F.IF.8	(Graph functions)				
	(Rate of change)	(Write a function)	MGSE9-12.F.IF.9				
	MGSE9-12.F.IF.7,7a,9	MGSE9-12.F.IF.8a,9	(Compare functions)				
	(Analyze functions)	(Compare & contrast functions)					
These units were written to build upon concepts from prior units, so later units contain tasks that depend upon the concepts addressed in earlier units.							

NOTE: Mathematical standards are interwoven and should be addressed throughout the year in as many different units and tasks as possible in order to stress the natural connections that exist among mathematical topics.

Grades 9-12 Key: Algebra Strand: SSE = Seeing Structure in Expressions, APR = Arithmetic with Polynomial and Rational Expressions, CED = Creating Equations, REI = Reasoning with Equations and Inequalities Functions Strand: IF = Interpreting Functions, LE = Linear and Exponential Models, BF = Building Functions, TF = Trigonometric Functions

**Geometry Strand:** CO = Congruence, SRT = Similarity, Right Triangles, and Trigonometry, C = Circles, GPE = Expressing Geometric Properties with Equations, GMD = Geometric Measurement and Dimension, MG = Modeling with Geometry

### **Cobb County School District**



Algebra I Teaching & Learning Framework							
Block Schedule							
Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6		
2 weeks	2.5 weeks	4.5 weeks	3 weeks	2.5 weeks	3.5 weeks		
Relationships	Reasoning with	Modeling & Analyzing Quadratic	Modeling & Analyzing	Comparing &	Describing Data		
Between	Linear Equations &	Functions	Exponential Functions	Contrasting			
Quantities &	Inequalities			Functions	Review & Extend		
Expressions							
MGSE9-12.N.RN.2-3 (Properties of rational & irrational numbers) MGSE9-12.N.Q.1-3 (Reason quantitatively & use units to solve problems) MGSE9-12.A.SSE.1 (Interpret expressions in context) MGSE9-12.A.SSE.1a-b (Interpret formulas & expressions in context) MGSE9-12.A.APR.1 (Add, subtract & multiply polynomials)	MGSE9-12.A.CED.1-4 (Create equations that describe numbers or relationships) MGSE9-12.A.REI.1,3,5 (Solve equations & inequalities 1-2 variable) MGSE9-12.A.REI.6 (Solve systems) MGSE9-12.A.REI.10-12 (Solve equations & inequalities 2 variables) MGSE9-12.F.BF.1 (Write a function) MGSE9-12.F.BF.1a,2 (Arithmetic & geometric sequences) MGSE9-12.F.IF.1 (Input vs. output) MGSE9-12.F.IF.2 (Function notation) MGSE9-12.F.IF.3-4 (Sequences & characteristics) MGSE9-12.F.IF.5-6 (Rate of change) MGSE9-12.F.IF.7,7a,9 (Analyze functions)	MGSE9-12.A.SSE.2 (Interpret the structure of expressions) MGSE9-12.A.SSE.3,3a-b (Equivalent forms of expressions) MGSE9-12.A.CED.1-2,4 (Create equations that describe numbers or relationships) MGSE9-12.A.REI.1 (Justify how to solve an equation) MGSE9-12.A.REI.4,4a-b (Methods of solving quadratics) MGSE9-12.F.BF.1,3 (Write a function & build new functions) MGSE9-12.F.IF.1 (Input vs. output) MGSE9-12.F.IF.2 (Function notation) MGSE9-12.F.IF.4 (Characteristics) MGSE9-12.F.IF.5-6 (Rate of change) MGSE9-12.F.IF.7,7a (Graph functions) MGSE9-12.F.IF.8 (Write a function) MGSE9-12.F.IF.8 (Write a function) MGSE9-12.F.IF.8a,9 (Compare & contrast functions)	MGSE9-12.A.CED.1-2 (Create equations 1-2 variables) MGSE9-12.A.REI.1 (Justify how to solve an equation) MGSE9-12.F.BF.1 (Write a function) MGSE9-12.F.BF.1a,2 (Arithmetic & geometric sequences) MGSE9-12.F.BF.3 (Build new functions) MGSE9-12.F.IF.1 (Input vs. output) MGSE9-12.F.IF.2 (Function notation) MGSE9-12.F.IF.3-4 (Sequences & characteristics) MGSE9-12.F.IF.5-6 (Rate of change) MGSE9-12.F.IF.7,7e (Graph functions) MGSE9-12.F.IF.9 (Compare functions)	MGSE9-12.F.LE.1 (Linear vs exponential) MGSE9-12.F.LE.1a (Growth of functions) MGSE9-12.F.LE.1b,c,2-3 (Changes in rate and relating to context) MGSE9-12.F.LE.5 (Interpret parameters) MGSE9-12.F.IF.3 (Build new functions) MGSE9-12.F.IF.1 (Input vs. output) MGSE9-12.F.IF.2 (Function notation) MGSE9-12.F.IF.4 (Characteristics) MGSE9-12.F.IF.5-6 (Rate of change) MGSE9-12.F.IF.7 (Graph functions) MGSE9-12.F.IF.9 (Compare functions)	MGSE9-12.S.ID.1 (Dot plots, histograms & box plots) MGSE9-12.S.ID.2 (Compare data distribution) MGSE9-12.S.ID.3 (Shape, center & spread) MGSE9-12.S.ID.5-6 (Bivariate data) MGSE9-12.S.ID.6a,c (Function of best fit) MGSE9-12.S.ID.7-9 (Slope, correlation coefficient, causation & correlation) <u>Review</u> : All standards by differentiating for student needs <u>Extend</u> : MGSE9-12.G.CO.1 (Precise definitions)		
	These units were written	to build upon concepts from prior units, so later All units will include the Mathematica	units contain tasks that depend u al Practices and indicate skills to m	pon the concepts addressed naintain	d in earlier units.		

NOTE: Mathematical standards are interwoven and should be addressed throughout the year in as many different units and tasks as possible in order to stress the natural connections that exist among mathematical topics.

Grades 9-12 Key: Algebra Strand: SSE = Seeing Structure in Expressions, APR = Arithmetic with Polynomial and Rational Expressions, CED = Creating Equations, REI = Reasoning with Equations and Inequalities Functions Strand: IF = Interpreting Functions, LE = Linear and Exponential Models, BF = Building Functions, TF = Trigonometric Functions

Geometry Strand: CO = Congruence, SRT = Similarity, Right Triangles, and Trigonometry, C = Circles, GPE = Expressing Geometric Properties with Equations, GMD = Geometric Measurement and Dimension, MG = Modeling with Geometry



Foundations of Algebra Teaching & Learning Framework							
Semester 1							
Module 1 2 weeks	Module 2 3 weeks	Module 3 3 weeks	Module 4 5 weeks	Module 5 5 weeks			
Number Sense	Arithmetic to	Proportional	Equations & Inequalities	Quantitative Reasoning with Functions			
& Quantity	Algebra	Reasoning					
MFANSQ1 Relationships (4 operations; multiples of fractions; multiple/divide by powers of ten with decimals; compare fractions/decimals) MFANSQ2 +/- (Real number+- on a number line & the meaning of zero) MFANSQ3 Irrationals (Irrational number approximations; adding & multiplying with rational & irrational numbers) MFANSQ4 Compute multi-digit decimals; compute with rational numbers; division of fractions by fractions; multi-step problems with any form of rational number) Algebra I Unit 1: MGSE9-12.N.RN.3 (Properties of rational & irrational numbers)	MFAAA1 Equivalent Expressions (Commutative & distributive properties; numerical & algebraic expressions; add, subtract & multiply algebraic expressions; equivalent expressions; evaluate formulas) MFAAA2 Exponents (integer exponents; formulas; square & cube roots; Pythagorean Theorem) Algebra I Unit 1: MGSE9-12.A.SSE.1 (Interpret expressions in context) MGSE9-12.A.SSE.1a-b (Interpret formulas & expressions in context) MGSE9-12.A.APR.1 (Add, subtract & multiply polynomials) MGSE9-12.N.RN.2 (Expressions with Radicals)	MFAPR1 Equivalent Ratios (Equivalent ratios) MFAPR2 Proportions (Fraction equivalence & division; percent problems) MFAPR3 Graphing (Unit rates as slope; similar triangles and slope; compare proportions in multiple representations) Algebra I Unit 1: MGSE9-12.N.Q.1-3 (Reason quantitatively & use units to solve problems)	MFAEI1 One Variable (Solve equations & inequalities & justify solutions) MFAEI2 Units (Scale, units, graphing) MFAEI3 Two Variables (Algebraic models; graphing calculators; systems of equations) MFAEI4 Literal Equations (Solve for a specific variable) Algebra I Unit 1: MGSE9-12.N.Q.1-3 (Reason quantitatively & use units to solve problems) Algebra I Unit 2: MGSE9-12.A.CED.1-4 (Create equations that describe numbers or relationships) MGSE9-12.A.REI.1,3,5 (Solve equations & inequalities 1-2 variable) MGSE9-12.A.REI.6 (Solve systems) MGSE9-12.A.REI.10-12 (Solve equations & inequalities 2 variables)	MFAQR1 Characteristics (Domain & range) MFAQR2 Compare & Graph (Rates of change; linear & non-linear; key features; compare with multiple representations) MFAQR3 Construct & Interpret (Write; variables in context; function notation) Algebra I Unit 2: MGSE9-12.F.BF.1 (Write a function) MGSE9-12.F.BF.1a,2 (Arithmetic sequences) MGSE9-12.F.IF.1 (Input vs. output) MGSE9-12.F.IF.2 (Function notation) MGSE9-12.F.IF.3-4 (Sequences & characteristics) MGSE9-12.F.IF.5-6 (Rate of change) MGSE9-12.F.IF.7,7a,9 (Analyze functions)			
	These units were written	to build upon concepts from pric	or units, so later units contain tasks that depend upon the c	concepts addressed in earlier units.			
NOTE: Mathematical stars	darde are intervenen and shawld	All units will include t	he Mathematical Practices and indicate skills to maintain	a natural connections that ovist among mathematical targing			
NSO number sonse & gu	TE: Mathematical standards are interwoven and should be addressed throughout the year in as many different units and tasks as possible in order to stress the natural connections that exist among mathematical topics						

NSQ- number sense & quantity

 $\ensuremath{\textbf{AA-}}\xspace$  arithmetic to algebra

PR- proportional reasoning

EI- equations and inequalities

QR- quantitative reasoning with functions



FA Algebra I Teaching & Learning Framework							
Semester 2							
Unit 3	Unit 4	Unit 5	Unit 6				
6 weeks	4 weeks	3 weeks	5 weeks				
Modeling & Analyzing Quadratic	Modeling & Analyzing Exponential	Comparing & Contrasting Functions	Describing Data				
Functions	Functions		Review and Extend				
MGSE9-12.A.SSE.2	MGSE9-12.A.CED.1-2	MGSE9-12.F.LE.1	MGSE9-12.S.ID.1				
(Interpret the structure of expressions)	(Create equations 1-2 variables)	(Linear vs exponential)	(Dot plots, histograms & box plots)				
MGSE9-12.A.SSE.3,3a-b	MGSE9-12.A.REI.1	MGSE9-12.F.LE.1a	MGSE9-12.S.ID.2				
(Equivalent forms of expressions)	(Justify how to solve an equation)	(Growth of functions)	(Compare data distribution)				
MGSE9-12.A.CED.1-2,4	MGSE9-12.F.BF.1	MGSE9-12.F.LE.1b,c,2-3	MGSE9-12.S.ID.3				
(Create equations that describe numbers	(Write a function)	(Changes in rate and relating to context)	(Shape, center & spread)				
or relationships)	MGSE9-12.F.BF.1a,2	MGSE9-12.F.LE.5	MGSE9-12.S.ID.5-6				
MGSE9-12.A.REI.1	(Arithmetic & geometric sequences)	(Interpret parameters)	(Bivariate data)				
(Justify how to solve an equation)	MGSE9-12.F.BF.3	MGSE9-12.F.BF.3	MGSE9-12.S.ID.6a,c				
MGSE9-12.A.REI.4,4a-b	(Build new functions)	(Build new functions)	(Function of best fit)				
(Methods of solving quadratics)	MGSE9-12.F.IF.1	MGSE9-12.F.IF.1	MGSE9-12.S.ID.7-9				
MGSE9-12.F.BF.1,3	(Input vs. output)	(Input vs. output)	(Slope, correlation coefficient, causation &				
(Write a function & build new functions)	MGSE9-12.F.IF.2	MGSE9-12.F.IF.2	correlation)				
MGSE9-12.F.IF.1	(Function notation)	(Function notation)					
(Input vs. output)	MGSE9-12.F.IF.3-4	MGSE9-12.F.IF.4	<b><u>Review</u>: All standards by differentiating</b>				
MGSE9-12.F.IF.2	(Sequences & characteristics)	(Characteristics)	for student needs				
(Function notation)	MGSE9-12.F.IF.5-6	MGSE9-12.F.IF.5-6					
MGSE9-12.F.IF.4	(Rate of change)	(Rate of change)	Extend:				
(Characteristics)	MGSE9-12.F.IF.7,7e	MGSE9-12.F.IF.7	MGSE9-12.G.CO.1				
MGSE9-12.F.IF.5-6	(Graph functions)	(Graph functions)	(Precise definitions)				
(Rate of change)	MGSE9-12.F.IF.9	MGSE9-12.F.IF.9					
MGSE9-12.F.IF.7,7a	(Compare functions)	(Compare functions)					
(Graph functions)							
MGSE9-12.F.IF.8							
(Write a function)							
MGSE9-12.F.IF.8a,9							
(Compare & contrast functions)							
These units were	These units were written to build upon concepts from prior units, so later units contain tasks that depend upon the concepts addressed in earlier units.						

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## Algebra 1 Unit 1: Relationships Between Quantities & Expression

### Volume 1 Issue 1

## References

### HMH Georgia Coordinate Algebra Text: Unit 1: Modules 1-2

Check with you teacher for online and print access:

Online website: my.hrw.com

### Web Resources

- Rational & irrational https://www.illustrative mathematics.org/conten t-standards/tasks/608
- Simplifying radicals http://cms.gavirtualscho ol.org/Shared/Math/GSE Algl16/GSEAlgI\_Relations hipsandExp\_Shared/GSE AlgI\_RelationshipsandExp \_Shared8.html#headingt aglink\_1
- Unit conversions

https://www.khanacade my.org/math/prealgebra/rates-andratios/metric-systemtutorial/v/unitconversion

 Polynomials http://mathbitsnotebook .com/Algebra1/Polynomi als/POoutline.html

### • Polynomials

http://www.brightstorm. com/search/?k=polynomi als

## **Dear Parents**

Below you will find a list of concepts that your child will use and understand while completing Unit 1: Relationships Between Quantities & Expressions. Also included are references, vocabulary and examples that will help you assist your child at home.

## Concepts Students will Use and Understand

- The structure of expressions and the meaning of their parts in context.
- Appropriateness of units of measure within context.
- Similarities between the system of polynomials and the system of integers.
- Addition, Subtraction, and Multiplication of polynomials is closed.
- Properties of rational and irrational numbers.
- Simplify and/or use the operations of addition, subtraction, and multiplication, with radicals within expressions limited to square roots.
- Visual representation of radicals.

## Vocabulary

- Binomial Expression: An algebraic expression with two unlike terms.
- **Capacity:** The greatest volume that a container can hold.
- **Coefficient:** A number multiplied by a variable.
- **Constant Term:** A quantity that does not change its value.
- **Factor:** When two or more integers are multiplied, each integer is a factor of the product. "To factor" means to write the number or term as a product of its factors.
- Irrational Number: A number whose decimal form is nonterminating and nonrepeating. Irrational numbers cannot be written in the form a/b, where a and b are integers (b cannot be zero). So all numbers that are not rational are irrational.
- Monomial Expression: An algebraic expression with one term.
- Polynomial function: A polynomial function is defined as a function,

 $f(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \dots + a_{n-2} x^2 + a_{n-1} x^1 + a_n$ , where the coefficients are real numbers.

- **Pythagorean Theorem:** It is a theorem that states a relationship that exists in any right triangle. If the lengths of the legs in the right triangle are *a* and *b* and the length of the hypotenuse is *c*, we can write the theorem as the following equation:  $a^2 + b^2 = c^2$
- **Radical:** The symbol,  $\sqrt[b]{a}$ , which is read "the bth root of a," is called a radical.
- **Radicand:** The number underneath the root symbol. So, in  $\sqrt[b]{a}$ , the *a* is called the radicand.
- **Rational Number:** A number expressible in the form *a/b* or *a/b* for some fraction *a/b*. The rational numbers include the integers.
- **Standard Form of a Polynomial**: To express a polynomial by putting the terms in descending exponent order.
- Term: A number, a variable, or a product of numbers and variables.
- **Trinomial**: An algebraic expression with three unlike terms.

## Algebra 1 Unit 1 Practice Problems

Example 1
A rectangle is 5m longer than it is wide. The perimeter is 38m. Find the length & width.
Example 2 Determine if $4 + \sqrt{7} = \frac{a}{b}$ is rational or irrational.
Example 3 What is the simplified form of $\sqrt{98}$ ?

Example 4 Find the difference. Write the answer in standard form.  $(-6x^3 + 5x - 3) - (2x^3 + 4x^2 - 3x + 1)$ 

### Example 5

A rectangle has a width of (x + 2) and a height of (2x + 1). Find an expression that represents the area as a whole.

## **Answer Key**

**Example 1** 2(w) + 2(w+5)=4w + 10; 4w + 10=38; w=7; the width is 7 and the length is 12

### Example 2

Irrational

Example 3  $7\sqrt{2}$ 

**Example 4**  $-8x^3 - 4x^2 + 8x - 4$ 

**Example 5**  $2x^2 + 5x + 2$ 



## Algebra I Unit 2 Reasoning with Linear Equations & Inequalities

### Volume 1 Issue 2

## References

#### HMH Georgia Coordinate Algebra Text:

Unit 1: Modules 1; Unit 2 Modules 3-5, 9-10, 12-13; Unit 3 Modules 8-9

Check with you teacher for online access: my.hrw.com

### Web Resources

- http://mathbitsnotebook.c om/Algebra1/LinearEquatio ns/LEGraphiLines.html
- http://mathbitsnotebook.c om/Algebra1/LinearEquatio ns/LEConstraintsLinearPract ice.html
- http://mathbitsnotebook.c om/Algebra1/Inequalities/I Qgraphinglinear2.html
- http://mathbitsnotebook.c om/Algebra1/Inequalities/I QGraphingPractice.html
- http://mathbitsnotebook.c om/Algebra1/Systems/SYlin earinequalities.html
- http://mathbitsnotebook.c om/Algebra1/Systems/SYGr aphIneqPractice.html
- http://mathbitsnotebook.c om/Algebra1/Functions/FN NotationEvaluation.html
- http://www.mathplay.com/slope-interceptgame.html
- http://www.webmath.com/ equline1.html
- http://www.mathplanet.co m/education/algebra-1/systems-of-linearequations-andinequalities/systems-oflinear-inequalities
- https://www.quia.com/rr/7 9715.html?AP\_rand=14742 76100
- http://www.purplemath.co m/modules/fcnops.htm

## **Dear Parents**

Below you will find a list of concepts that your child will use and understand while completing Unit 2: Reasoning with Linear Equations & Inequalities. Also included are references, vocabulary and examples that will help you assist your child at home.

## Concepts Students will Use and Understand

- Create Equations that Describe Numbers or Relationships
- Solve Equations & Inequalities
- Build a Function that Models a Relationship Between Two Quantities
  - Understand the Concept of Function & Use Function Notation
- Interpret Functions that Arise in Applications in Terms of Context
- Analyze Functions using Different Representations

## Vocabulary

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- Arithmetic Sequence. A sequence of numbers in which the difference between any two consecutive terms is the same.
- Average Rate of Change. The change in the value of a quantity by the elapsed time. For a function, this is the change in the *y*-value divided by the change in the *x*-value for two distinct points on the graph.
- **Constant Rate of Change.** With respect to the variable x of a linear function y = f(x), the constant rate of change is the slope of its graph.
- **Continuous.** Describes a connected set of numbers, such as an interval.
- **Discrete.** A set with elements that are disconnected.
- **Domain.** The set of *x*-coordinates of the set of points on a graph; the set of *x*-coordinates of a given set of ordered pairs. The value that is the input in a function or relation.
- End Behaviors. The appearance of a graph as it is followed farther and farther in either direction.
- **Explicit Formula.** A formula that allows direct computation of any term for a sequence *a*1, *a*2, *a*3, ..., *an*, ...
- **Factor.** For any number *x*, the numbers that can be evenly divided into *x* are called factors of *x*. For example, the number 20 has the factors 1, 2, 4, 5, 10, and 20.
- Interval Notation. A notation representing an interval as a pair of numbers. The numbers are the endpoints of the interval. Parentheses and/or brackets are used to show whether the endpoints are excluded or included.
- Linear Function. A function with a constant rate of change and a straight line graph.
- Linear Model. A linear function representing real-world phenomena. The model also represents patterns found in graphs and/or data.
- **Parameter.** The independent variable or variables in a system of equations with more than one dependent variable.
- Range. The set of all possible outputs of a function.
- **Recursive Formula.** A formula that requires the computation of all previous terms to find the value of *an*.

- Slope. The ratio of the vertical and horizontal changes between two points on a surface or a line.
- X-intercept. The point where a line meets or crosses the x-axis
- **Y-intercept.** The point where a line meets or crosses the y-axis

### **Algebra 1 Unit 2 Practice Problems**

### Formulas

y = mx + b

Slope-Intercept:

### Example 1

The sum of two consecutive integers is less than 83. Find the pair of integers with the greatest sum.

### Example 2

**Arithmetic Sequence:** 

 $A_n = a_1 + (n-1)d$ 

Pablo and his family are driving to California for vacation. The trip is 1,505 miles and they drive at an average speed of 59 mph. Which equation would give the number of miles remaining until they reach their destination, M, in terms of h, the number of hours they have driven?

A. M = 59 + 1,505hB. M = 1,505 - 59hC. M = 1,505 + 59hD. M = 59 - 1,505h

### **Example 3**

Britany is leaving for an 800 mile road trip. Her plan is not to make any stops until she has 590 miles, or less, left of the drive. She is averaging 70 miles per hour. If *x* represents the number of hours driving, which of the following inequalities symbolizes this situation?

A. 590 - 70x > 800	B. 800 - 70x < 590
C. 590 - 70x < 800	D. 800 - 70x > 590

### Example 4

What is the next term in this sequence? 4, 10, 16, ...

### Example 5

Generate ordered pairs for the function y = x + 3 for x = -2, -1, 0, 1, and state the domain and range.

### **Answer Key**

### Example 1

Define a Variable: x = the first consecutive number, so x +1 = the second consecutive number Equation: x + x + 1 < 83 2x < 82 x < 41

The numbers are 40 and 41 Check: 40 + 41 < 83 81 < 83

### Example 2

B. M = 1,505 - 59h Example 3 B. 800 - 70x < 590 Example 4 22 Example 5 (-2,1), (-1,2), (0,3), (1,4), (2,5) Domain: {-2,-1,0,1,2} Range: {1,2,3,4,5})



## Algebra 1 Unit 3: Modeling & Analyzing Quadratic Functions

### Volume 1 Issue 3

## References

HMH Georgia Analytic Geometry Text: Unit 5: Modules 14-16

Check with you teacher for online and print access:

> Online website: my.hrw.com

### Web Resources

- http://www.purplemath
   .com/modules/quadfor
   m.htm (quadratic
   equation)
- http://www.purplemath .com/modules/solvquad
   .htm (solving quadratic equations)
- http://www.purplemath
   .com/modules/grphqua
   d2.htm (vertex form)
- http://www.analyzemat
   h.com/quadratics/quadr
   atics.htm (standard
   form)
- http://www.purplemath .com/modules/ineqqua
   d.htm (quadratic inequalities)

## **Dear Parents**

Below you will find a list of concepts that your child will use and understand while completing Unit 3: Modeling & Analyzing Quadratic Functions. Also included are references, vocabulary and examples that will help you assist your child at home.

## Concepts Students will Use and Understand

Students will analyze quadratic functions in the forms  $f(x) = ax^2 + bx + c$  and  $f(x) = a(x - h)^2 + k$ 

- Convert between standard and vertex form
  - Graph quadratic functions as transformations of  $f(x) = x^2$
- Investigate & explain characteristics of quadratic functions
- Explore quadratic sequences recursively and explicitly
- Students will solve quadratic equations and inequalities in one variable.
- Solve equations graphically & with technology
- Find real & complex solutions of equations by factoring, taking square roots, and applying the quadratic formula
- Analyze roots using technology and the discriminant
- Solve quadratic inequalities both graphically and algebraically
- Describe the solutions using linear inequalities and interval notation

## Vocabulary and Theorems

- **Complete factorization over the integers.** Writing a polynomial as a product of polynomials so that none of the factors is the number 1, there is at most one factor of degree zero, each polynomial factor has degree less than or equal to the degree of the product polynomial, each polynomial factor has all integer coefficients, and none of the factor polynomial can written as such a product.
- **Completing the square**. Completing the Square is the process of converting a quadratic equation into a perfect square trinomial by adding or subtracting terms on both sides.
- **Difference of two squares**. A squared (multiplied by itself) number subtracted from another squared number. It refers to the identity  $a^2 b^2 = (a + b)(a b)$  in elementary algebra.
- **Discriminant of a quadratic equation.** The discriminant of a quadratic equation of the form

 $ax^2 + bx + c = 0$ ,  $a \neq 0$ , is the number  $b^2 - 4ac$ .

- Horizontal shift. A rigid transformation of a graph in a horizontal direction, either left or right.
- Perfect square trinomial. A trinomial that factors into two identical binomial factors.
- Quadratic equation. An equation of degree 2, which has at most two solutions.
- Quadratic function. A function of degree 2 which has a graph that "turns around" once, resembling an umbrella–like curve that faces either right–side up or upside down. This graph is called a parabola.
- **Root.** The *x*-values where the function has a value of zero.
- Standard form of a quadratic function.  $ax^2 + bx + c$

parabola is opening up or down, or in terms of x if the parabola is opening left or right.

······

Vertex form of a quadratic function. A formula for a quadratic equation of the form f

 (x) = a (x - h)<sup>2</sup> + k, where a is a nonzero constant and the vertex of the graph is the point (h, k).

### Theorems

For 
$$h = \frac{-b}{2a}$$
 and  $k = f\left(\frac{-b}{2a}\right)$ ,  $f(x) = a(x-h)^2 + k$  is the same function as  $f(x) = ax^2 + bx + c$ .

The graph of any quadratic function can be obtained from transformations of the graph of the basic function  $f(x) = x^2$ .

**Quadratic formula**: The solution(s) of the quadratic equation of the form  $ax^2 + bx + c = 0$ , where a, b, and c are real numbers with  $a \neq 0$ , is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The discriminant of a quadratic equation is positive, zero, or negative if and only if the equation has two real solutions, one real solution, or two complex conjugate number solutions respectively.

### **Algebra 1 Unit 3 Practice Problems**

Formulas Example 1 What happens to the graph of  $y = x^2$  when you multiply  $x^2$  by 3? **Quadratic Equations:** Standard Form: Example 2 Find the zeros of (2x + 3)(3x + 4) = 0.  $y = ax^2 + bx + c$ Example 3 Factor 6x<sup>2</sup> +7x -20 Vertex Form: Example 4  $y = a(x - h)^{2} + k$ Solve the quadratic equation:  $2x^2 + 3x - 54 = 0$ . Example 5 **Quadratic Formula:** Find the y-intercept of  $f(x) = x^2 - 4x + 9$ .  $-b \pm \sqrt{b^2 - 4ac}$ Example 6 Find the discriminate of  $3x^2 + 15x = 12$ **Answer Key** Example 1 It causes a vertical stretch. (see graph to right) 20Example 2 x = -3/2 or x = -4/3Example 3 15 (2x + 5)(3x - 4)Example 4 (2x-9)(x+6) = 010x = 4.5 or x= -6  $y = x^2$ Example 5  $f(0) = 0^2 - 4(0) + 9$ y-intercept = 9 Example 6 b<sup>2</sup>-4ac = (15)<sup>2</sup>-4(-36) = 369



## Algebra 1 Unit 4: Modeling & Analyzing Exponential Functions

### Volume 1 Issue 4

## References

### HMH Georgia Coordinate Algebra Text:

Unit 3: Modules 12-13

# Check with you teacher for online and print access:

Online website: my.hrw.com

### Web Resources

• Graphing Exponential Functions http://cms.gavirtualschool.org/Shar ed/Math/GSEAlgl16/GSEAlgl\_Propof ExpandGraphExpFunc\_Shared/GSEAl gl\_PropofExpandGraphExpFunc\_Sha red4.html

### • Characteristic of Exponential Functions

http://cms.gavirtualschool.org/Shar ed/Math/GSEAlgl16/GSEAlgl\_Propof ExpandGraphExpFunc\_Shared/GSEAl gl\_PropofExpandGraphExpFunc\_Sha red7.html

### • Exponential Functions

http://www.purplemath.com/modul es/expofcns.htm

• Exponential functions

https://mathbitsnotebook.com/Alge bra1/FunctionGraphs/FNGTypeExpo nential.html

### Exponential functions

http://www.mathsisfun.com/sets/function-exponential.html

Geometric sequences
 http://mathbitsnotebook.com/Algeb
 rot (Supprises (ENG any page http://

ra1/Functions/FNSequences.html
Geometric sequences http://www.purplemath.com/modul es/series3.htm

### Geometric sequences

http://www.basicmathematics.com/geometricsequence.html

### **Dear Parents**

Below you will find a list of concepts that your child will use and understand while completing Unit 4: Modeling & Analyzing Exponential Functions. Also included are references, vocabulary and examples that will help you assist your child at home.

### **Concepts Students will Use and Understand**

- Analyze & interpret exponential functions in real-world applications.
- Build on and informally extend understanding of integer exponents to consider exponential functions.
- Use function notation.
- Interpret expressions for functions in terms of the situation they model.
- Analyze exponential functions and model how different representations may be used based on the situation presented.
- Build a function to model a relationship between two quantities.
- Recognize geometric sequences as exponential functions.
- Create new functions from existing functions.
- Construct and compare exponential models and solve problems.
- Reinforce their previous understanding of characteristics of graphs and investigate key features of exponential graphs.
- Investigate a multiplicative change in exponential functions.
- Create and solve exponential equations.
- Apply related linear equations solution techniques and the laws of exponents to the creation and solution of simple exponential equations.

### Vocabulary

- **Explicit Expression.** A formula that allows direct computation of any term for a sequence *a*<sub>1</sub>, *a*<sub>2</sub>, *a*<sub>3</sub>, ..., *a*<sub>n</sub>, ....
- **Exponential Function.** A nonlinear function in which the independent value is an exponent in the function, as in  $y = ab^x$ .
- **Exponential Model.** An exponential function representing real-world phenomena. The model also represents patterns found in graphs and/or data.
- **Geometric Sequence.** A sequence of numbers in which the ratio between any two consecutive terms is the same. In other words, you multiply by the same number each time to get the next term in the sequence. This fixed number is called the common ratio for the sequence.
- **Recursive Formula.** A formula that requires the computation of all previous terms to find the value of *a<sub>n</sub>*.

Exponential Growth & Decay	Compound Interest
Growth:     Decay: $y = a(1+r)^x$ $y = a(1-r)^x$ $a$ = initial amount before measuring growth/decay $r$ = growth/decay rate (often a percent) $x$ = number of time intervals that have passed	$\begin{split} A &= P\left(1+\frac{r}{n}\right)^{nt}\\ \text{Where,}\\ &= P = \text{principal amount (initial investment)}\\ &= r = \text{annual nominal interest rate (as a decimal)}\\ &= n = \text{number of the the interest is compounded per yeal}\\ &= t = \text{number of years} \end{split}$
Geometric Sequence: Recursive $a_n = a_{n-1}r$	Geometric Sequence: Explicit $a_n = a_1 r^{n-1}$

## Algebra 1 Unit 4 Practice Problems

### Formulas

### **Exponential Growth/Decay:**



### **Compound Interest:**

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$
  
Where,  
• P = principal amount (initial investment)  
• r = annual nominal interest rate (as a decimal)  
• n = number of times the interest is compounded per year  
• t = number of years

#### **Geometric Sequences:**

### Recursive

$$a_n = a_{n-1}r$$

#### Explicit



1.) Tell whether each set of ordered pairs satisfies an exponential function. Explain your answer.

 $\{(-1, 1), (0, 0), (1, 1), (2, 4)\}$ 

Yes; as the x-values change by a constant amount, the y-values are multiplied by a constant amount.

2.) Choose several values of x and generate ordered pairs. Then use the ordered pairs to graph the function:  $y = 0.2(5^x)$ 



x	у
<mark>-1</mark>	<mark>.04</mark>
<mark>0</mark>	<mark>.2</mark>
<mark>1</mark>	1
2	<mark>5</mark>

3.) In the definition of an exponential function, the value of *b* cannot be 1, and the value of *a* cannot be 0. Why?

If the value of *b* were 1, the function would be constant. If the value of *a* were 0, the function would be the constant function of y = 0.

#### 4.) Technology Application:

Moore's law states that the maximum number of transistors that can fit on a silicon chip doubles every two years. The function  $f(x) = 42(1.41)^x$  models the number of transistors, in millions, that can fit on a chip, where x is the number of years since 2000. Using this model, in what year can a chip hold 1 billion transistors?

#### About 2009

5.) The population of a town is decreasing at a rate of 3% per year. In 2000, there were 1700 people. Write an exponential decay function to model this situation. Then find the population in 2012.

 $\frac{y = 1700(0.97)^t}{\text{Population: } 1180 \text{ people}}$ 



## <u>Algebra I</u> Unit 5: Comparing & Contrasting Functions

### Volume 1 Issue 5

## References

### HMH Georgia Advanced Algebra Text:

Unit 3: Module 13

# Check with you teacher for online and print access:

Online website: my.hrw.com

### Web Resources

### • Rate of Change

http://www.nms.org/Portals/0/D ocs/FreeLessons/Fill%20lt%20Up ,%20Please%20-%20Part%20Up.pdf

%20Part%20III.pdf

• Distinguishing between Linear & Exponential Functions

https://learnzillion.com/lesson\_p lans/6663/

• Comparing Graphs of Linear, Quadratic, & Exponential Functions

http://www.virtualnerd.com/alg ebra-1/quadratic-equationsfunctions/linear-exponentialcomparison/linear-exponentialcomparison-graphingexamples/determine-functiontype-from-graph

### **Dear Parents**

Below you will find a list of concepts that your child will use and understand while completing Unit 5: Comparing & Contrasting Functions. Also included are references, vocabulary and examples that will help you assist your child at home.

### **Concepts Students will Use and Understand**

- Deepen their understanding of linear, quadratic, and exponential functions as they compare and contrast the three types of functions.
- Understand the parameters of each type of function in contextual situations.
- Interpret linear, quadratic, and exponential functions that arise in applications in terms of the context.
- Analyze linear, quadratic, and exponential functions and model how different representations may be used based on the situation presented.
- Construct and compare characteristics of linear, quadratic, and exponential models and solve problems.
- Distinguish between linear, quadratic, and exponential functions graphically, using tables, and in context.
- Recognize that exponential and quadratic functions have a variable rate of change while linear functions have a constant rate of change.
- Distinguish between additive and multiplicative change and construct and interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.
- Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

### Vocabulary

All vocabulary is repeated from units 2-4.



## **Unit 5 Practice Problems**

- Answers
- 1.) <mark>Exponential</mark>



2.) Linear: Constant Quadratic: Variable Exponential: Variable (Average rate of change will eventually be greater than quadratic or linear rates of change.)

3.) Quadratic Function

4.) Plan A: y = 50x + 500Plan B:  $y = 500(1.05)^{x}$ 



More homes will be built under plan A up to the end of the 26<sup>th</sup> year. After that, more homes will be built under plan B, and plan B results in more homes than plan A by ever-increasing amounts each year. 1.) Graph the set of data. Which kind of model best describes the data?  $\{(-1, 4), (-2, 0.8), (0, 20), (1, 100), (-3, 0.16)\}$ 

2.) Describe the rate of change for linear, quadratic, & exponential functions.

3.) Determine which function model the data in the table represents:

Height of Bridge Suspension Cables				
Cable's Distance Cable's Heig				
from Tower (ft)	(ft)			
0	400			
100	256			
200	144			
300	64			

4.) A town home has approximately 500 homes. The town council is considering plans for future development. Plan A calls for an increase of 50 homes per year. Plan B calls for a 5% increase each year. Compare the plans.



## <u>Algebra I</u> Unit 6: Describing Data

### Volume 1 Issue 6

## References

HMH Georgia Coordinate Algebra Text: Unit 4: Modules 14 & 15 HMH Georgia Analytic Geometry Text: Unit 5: Module 15.3 (quadratic regression)

### Check with you teacher for online and print access:

Online website: my.hrw.com

### Web Resources

### GA Virtual: Interpreting and Representing Two Variable Data

http://cms.gavirtualschool.org/S hared/Math/GSEAlgI16/GSEAlgI\_ IntandRepTwoVarData\_Shared/i ndex.html

- Khan Academy https://www.khanacademy.org/mat h/probability/regression
- Correlation Coefficient http://mathbits.com/MathBits/TI Section/Statistics2/correlation.ht m
- Two-way Frequency Tables

https://mathbitsnotebook.com/Alge bra1/StatisticsReg/ST2TwoWayTable .html

• Shapes of Distributions

http://www.mathbitsnotebook.c om/Algebra1/StatisticsData/STSh apes.html

### **Dear Parents**

Below you will find a list of concepts that your child will use and understand while completing Unit 6: Describing Data. Also included are references, vocabulary and examples that will help you assist your child at home.

### **Concepts Students will Use and Understand**

- Know how to compute the mean, median, interquartile range, and mean absolute deviation by hand in simple cases and using technology with larger data sets.
- Find the lower extreme (minimum), upper extreme (maximum), and quartiles.
- Use and interpret shape, center, and spread of data.
- Create a graphical representation of a data set.
- Summarize data in two-way frequency table.
- Represent data in a scatter plot and describe how the variables are related.
- Interpret the slope & y-intercept of a line from any representation.
- Find linear, quadratic, and exponential regressions.
- Compute and interpret the correlation coefficient.
- Understand the meaning of correlation and causation.

### Vocabulary

- **Bivariate data.** Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.
- **Conditional Frequencies.** The relative frequencies in the body of a two-way frequency table.
- **Correlation Coefficient.** A measure of the strength of the linear relationship between two variables that is defined in terms of the (sample) covariance of the variables divided by their (sample) standard deviations.
- Joint Frequencies. Entries in the body of a two-way frequency table.
- **Marginal Frequencies.** Entries in the "Total" row and "Total" column of a two-way frequency table.
- Mean Absolute Deviation. A measure of variation in numerical data by adding the distance between each data point and the mean, then dividing by the number of values
- **Shape**. The shape of a distribution is described by symmetry, number of peaks, direction of skew, or uniformity.
- **Symmetry**. A symmetric distribution can be divided at the center so that each half is a mirror image of the other.
- **Number of Peaks.** Distributions can have few or many peaks. Distributions with one clear peak are called unimodal and distributions with two clear peaks are called bimodal. Unimodal distributions are sometimes called bell-shaped.
- **Direction of Skew**. Some distributions have many more observations on one side of graph than the other. Distributions with a tail on the right toward the higher values are said to be skewed right; and distributions with a tail on the left toward the lower values are said to be skewed left.
- **Uniformity-** When observations in a set of data are equally spread across the range of the distribution, the distribution is called uniform distribution. A uniform distribution has no clear peaks.
- Trend. A change (positive, negative or constant) in data values over time.

## **Practice Problems**



 When you look at the shape of the data, if the "long tail" is on the left=skewed left, if it is on the right=skewed right, and if it is evenly distributed it is symmetric.



3.) The correlation coefficient is approximately .999. This means the line of best fit is extremely accurate because the coefficient is so close to 1.

4.) table in yellow at bottom is the key
a. 36%, 19%, 45%, 54%, 46%
b. 25%, 9%, 20%(top), 11%, 10%,
25%(bottom)
c. 54%
d. 36%

e. <mark>69.4%</mark>

1.) Find the linear regression of the following data:

Fahrenheit degrees (°F)	Celsius degrees (°C)
32	0
68	20
86	30
122	50
158	70
194	90
212	100

2.) Explain when data is skewed left, right, or symmetric.

3.) Using technology, determine the correlation coefficient. Interpret its meaning. (0,20) (1,40) (2,75) (3,150) (4, 297) (5,510)

4.) Construct a frequency table from the following information: A survey of 200 9th and 10th graders was given to determine what their favorite subject was. 72 said Math (50 which were freshmen), 38 said Social Studies (20 which were sophomores), and 40 freshmen and 50 sophomores said PE was their favorite.

Based on your tables above, answer the following questions:

- a. What are the marginal relative frequencies?
- b. What are the joint relative frequencies?
- c. What is the probability that a student surveyed is a freshman?
- d. What is the probability that a student surveyed likes Math?
- e. If a student likes Math, what is the probability that they are a freshman?

	Math	SS	PE	Total
9th	50	18	40	108
10th	22	20	50	92
Total	72	38	90	200



Algebra II Teaching & Learning Framework						
Semester 1 Semester 2						
Unit 1 4 weeks	Unit 2 5 weeks	Unit 3 6 weeks	Unit 4 6 weeks	Unit 5 5 weeks	Unit 6 5 weeks	Unit 7 5 weeks
Quadratics Revisited	Operations with Polynomials	Polynomial Functions	Rational & Radical Relationships	Exponentials & Logarithms	Mathematical Modeling	Inferences and Conclusions from Data Review & Extend
MGSE9-12.N.CN.1 (Complex numbers) MGSE9-12.N.CN.2 (Complex numbers & properties) MGSE9-12.N.CN.3 (Conjugate of complex numbers) MGSE9-12.N.CN.7 (Solve quadratics with complex solutions) MGSE9-12.N.CN.8 (Factoring with complex solutions) MGSE9-12.A.REI.4 (Solve quadratics in 1 variable) MGSE9-12.A.REI.4b (Solve quadratic equations by inspection) MGSE9-12.N.RN.1 (Rational exponents) MGSE9-12.N.RN.2 (Expressions with radicals & rational exponents)	MGSE9-12.A.APR.1 (Add, subtract & multiply polynomials) MGSE9-12.A.APR.5 (Binomial Theorem) MGSE9-12.A.APR.6 (Rewrite rational expressions) MGSE9-12.F.BF.1 (Write a function) MGSE9-12.F.BF.1b (Combine standard functions) MGSE9-12.F.BF.1c (Compose functions) MGSE9-12.F.BF.4 (Inverse functions) MGSE9-12.F.BF.4a (f(x)=c & inverse) MGSE9-12.F.BF.4b (Use composition to verify inverses) MGSE9-12.F.BF.4c (Values of inverse function from graph or table)	MGSE9-12.N.CN.9 (Fundamental Theorem of Algebra) MGSE9-12.A.SSE.1,a,b (Interpret expressions; Interpret parts & terms of expressions) MGSE9-12.A.SSE.2 (Equivalent expressions) MGSE9-12.A.APR.2 (Remainder Theorem) MGSE9-12.A.APR.3 (Identify zeros) MGSE9-12.A.APR.4 (Polynomial Identities) MGSE9-12.F.IF.4 (Characteristics of functions) MGSE9-12.F.IF.7 (Graph functions) MGSE9-12.F.IF.7c (Graph polynomial functions)	MGSE9-12.A.APR.7 (Rewrite rational expressions) MGSE9-12.A.CED.1 (Create equations & inequalities-1 variable) MGSE9-12.A.CED.2 (create equations & inequalities-2 variables) MGSE9-12.A.REI.2 (Solve simple radical & rational equations) MGSE9-12.F.IF.4 (Characteristics of functions) MGSE9-12.F.IF.7 (Domains of functions) MGSE9-12.F.IF.7 (Graph Functions) MGSE9-12.F.IF.7b (Graph square rt, cube rt, piecewise, step & absolute value functions) MGSE9-12.F.IF.7d (Graph rational functions)	MGSE9-12.A.SSE.3 (Equivalent expressions) MGSE9-12.A.SSE.3c (Properties of exponents) MGSE9-12.F.IF.7 (Graph functions) MGSE9-12.F.IF.7e (Graph exponential & logarithmic functions) MGSE9-12.F.IF.8 (Write a function) MGSE9-12.F.IF.8b (Interpret expressions) MGSE9-12.F.BF.5 (Inverse relationships) MGSE9-12.F.LE.4 (Express exponential models as logarithmic)	MGSE9-12.A.SSE.4 (Derive formula for sum of finite geometric series) MGSE9-12.A.CED.1 (Create equations & inequalities-1 variable) MGSE9-12.A.CED.2 (create equations & inequalities-2 variables) MGSE9-12.A.CED.3 (Represent constraints) MGSE9-12.A.CED.4 (Rearrange formulas) MGSE9-12.A.REI.11 (Solutions to equations) MGSE9-12.F.IF.6 (Average rate of change) MGSE9-12.F.IF.9 (Compare 2 functions) MGSE9-12.F.BF.3 (Build new functions from existing functions)	MGSE9-12.S.ID.2 (Shape & data distribution) MGSE9-12.S.ID.4 (Fit to a normal distribution) MGSE9-12.S.IC.1 (Inferences from a random sample) MGSE9-12.S.IC.2 (Using simulations) MGSE9-12.S.IC.3 (Randomization) MGSE9-12.S.IC.4 (Population mean) MGSE9-12.S.IC.5 (Compare 2 treatments) MGSE9-12.S.IC.6 (Evaluate reports based on data) <u>Review:</u> All standards by differentiating for student needs <u>Extend:</u> MGSE9-2.N.CN.4 (Complex plane)
These units were written to build upon concepts from prior units, so later units contain tasks that depend upon the concepts addressed in earlier units. All units will include the Mathematical Practices and indicate skills to maintain.						

**NOTE:** Mathematical standards are interwoven and should be addressed throughout the year in as many different units and tpmmasks as possible in order to stress the natural connections that exist among mathematical topics.

Grades 9-12 Key: Algebra Strand: SSE = Seeing Structure in Expressions, APR = Arithmetic with Polynomial and Rational Expressions, CED = Creating Equations, REI = Reasoning with Equations and Inequalities Functions Strand: IF = Interpreting Functions, LE = Linear and Exponential Models, BF = Building Functions, TF = Trigonometric Functions

Geometry Strand: CO = Congruence, SRT = Similarity, Right Triangles, and Trigonometry, C = Circles, GPE = Expressing Geometric Properties with Equations, GMD = Geometric Measurement and Dimension, MG = Modeling with Geometry



Algebra II Teaching & Learning Framework							
Block Schedule							
Unit 1 2 weeks	Unit 2 2.5 weeks	Unit 3 3 weeks	Unit 4 3 weeks	Unit 5 2.5 weeks	Unit 6 2.5 weeks	Unit 7 2.5 weeks	
Quadratics Revisited	Operations with Polynomials	Polynomial Functions	Rational & Radical Relationships	Exponentials & Logarithms	Mathematical Modeling	Inferences and Conclusions from Data Review & Extend	
MGSE9-12.N.CN.1 (Complex numbers) MGSE9-12.N.CN.2 (Complex numbers & properties) MGSE9-12.N.CN.3 (Conjugate of complex numbers) MGSE9-12.N.CN.7 (Solve quadratics with complex solutions) MGSE9-12.N.CN.8 (Factoring with complex solutions) MGSE9-12.A.REI.4 (Solve quadratics in 1 variable) MGSE9-12.A.REI.4b (Solve quadratic equations by inspection) MGSE9-12.N.RN.1 (Rational exponents) MGSE9-12.N.RN.2 (Expressions with radicals & rational exponents)	MGSE9-12.A.APR.1 (Add, subtract & multiply polynomials) MGSE9-12.A.APR.5 (Binomial Theorem) MGSE9-12.A.APR.6 (Rewrite rational expressions) MGSE9-12.F.BF.1 (Write a function) MGSE9-12.F.BF.1b (Combine standard functions) MGSE9-12.F.BF.1c (Compose functions) MGSE9-12.F.BF.4 (Inverse functions) MGSE9-12.F.BF.4a (f(x)=c & inverse) MGSE9-12.F.BF.4b (Use composition to verify inverses) MGSE9-12.F.BF.4c (Values of inverse function from graph or table)	MGSE9-12.N.CN.9 (Fundamental Theorem of Algebra) MGSE9-12.A.SSE.1,a,b (Interpret expressions; Interpret parts & terms of expressions) MGSE9-12.A.SSE.2 (Equivalent expressions) MGSE9-12.A.APR.2 (Remainder Theorem) MGSE9-12.A.APR.3 (Identify zeros) MGSE9-12.A.APR.4 (Polynomial Identities) MGSE9-12.F.IF.4 (Characteristics of functions) MGSE9-12.F.IF.7 (Graph functions) MGSE9-12.F.IF.7c (Graph polynomial functions)	MGSE9-12.A.APR.7 (Rewrite rational expressions) MGSE9-12.A.CED.1 (Create equations & inequalities-1 variable) MGSE9-12.A.CED.2 (create equations & inequalities-2 variables) MGSE9-12.A.REI.2 (Solve simple radical & rational equations) MGSE9-12.F.IF.4 (Characteristics of functions) MGSE9-12.F.IF.7 (Domains of functions) MGSE9-12.F.IF.7 (Graph Functions) MGSE9-12.F.IF.7b (Graph square rt, cube rt, piecewise, step & absolute value functions) MGSE9-12.F.IF.7d (Graph rational functions)	MGSE9-12.A.SSE.3 (Equivalent expressions) MGSE9-12.A.SSE.3c (Properties of exponents) MGSE9-12.F.IF.7 (Graph functions) MGSE9-12.F.IF.7e (Graph exponential & logarithmic functions) MGSE9-12.F.IF.8 (Write a function) MGSE9-12.F.IF.8b (Interpret expressions) MGSE9-12.F.IBF.5 (Inverse relationships) MGSE9-12.F.LE.4 (Express exponential models as logarithmic)	MGSE9-12.A.SSE.4 (Derive formula for sum of finite geometric series) MGSE9-12.A.CED.1 (Create equations & inequalities-1 variable) MGSE9-12.A.CED.2 (create equations & inequalities-2 variables) MGSE9-12.A.CED.3 (Represent constraints) MGSE9-12.A.CED.4 (Rearrange formulas) MGSE9-12.A.REI.11 (Solutions to equations) MGSE9-12.F.IF.6 (Average rate of change) MGSE9-12.F.IF.9 (Compare 2 functions) MGSE9-12.F.BF.3 (Build new functions from existing functions)	MGSE9-12.S.ID.2 (Shape & data distribution) MGSE9-12.S.ID.4 (Fit to a normal distribution) MGSE9-12.S.IC.1 (Inferences from a random sample) MGSE9-12.S.IC.2 (Using simulations) MGSE9-12.S.IC.3 (Randomization) MGSE9-12.S.IC.3 (Randomization) MGSE9-12.S.IC.4 (Population mean) MGSE9-12.S.IC.5 (Compare 2 treatments) MGSE9-12.S.IC.6 (Evaluate reports based on data) <u>Review:</u> All standards by differentiating for student needs <u>Extend:</u> MGSE9-2.N.CN.4 (Complex plane)	
These units were written to build upon concepts from prior units, so later units contain tasks that depend upon the concepts addressed in earlier units. All units will include the Mathematical Practices and indicate skills to maintain.							

**NOTE:** Mathematical standards are interwoven and should be addressed throughout the year in as many different units and tpmmasks as possible in order to stress the natural connections that exist among mathematical topics.

Grades 9-12 Key: Algebra Strand: SSE = Seeing Structure in Expressions, APR = Arithmetic with Polynomial and Rational Expressions, CED = Creating Equations, REI = Reasoning with Equations and Inequalities Functions Strand: IF = Interpreting Functions, LE = Linear and Exponential Models, BF = Building Functions, TF = Trigonometric Functions

Geometry Strand: CO = Congruence, SRT = Similarity, Right Triangles, and Trigonometry, C = Circles, GPE = Expressing Geometric Properties with Equations, GMD = Geometric Measurement and Dimension, MG = Modeling with Geometry

### **Cobb County School District**

Honors Algebra II Teaching & Learning Framework							
	Semes	ster 1			Semester 2		
Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	
4 weeks	5 weeks	o weeks	0 weeks	5 weeks	5 weeks	5 weeks	
Quadratics	<b>Operations with</b>	Polynomial Functions	Rational & Radical	Exponentials &	Mathematical	Inferences and	
Revisited	Polynomials		Relationships	Logarithms	Modeling	Conclusions from Data Review & Extend	
MGSE9-12.N.CN.1 (Complex numbers) MGSE9-12.N.CN.2 (Complex numbers & properties) MGSE9-12.N.CN.3 (Conjugate of complex numbers) MGSE9-12.N.CN.7 (Solve quadratics with complex solutions) MGSE9-12.N.CN.8 (Factoring with complex solutions) MGSE9-12.A.REI.4 (Solve quadratic in 1 variable) MGSE9-12.A.REI.4b (Solve quadratic equations by inspection) MGSE9-12.N.RN.1 (Rational exponents) MGSE9-12.N.RN.2 (Expressions with radicals & rational exponents)	MGSE9-12.A.APR.1 (Add, subtract & multiply polynomials) MGSE9-12.A.APR.5 (Binomial Theorem) MGSE9-12.A.APR.6 (Rewrite rational expressions) MGSE9-12.F.BF.1 (Write a function) MGSE9-12.F.BF.1b (Combine standard functions) MGSE9-12.F.BF.1c (Compose functions) MGSE9-12.F.BF.4 (Inverse functions) MGSE9-12.F.BF.4a (f(x)=c & inverse) MGSE9-12.F.BF.4b (Use composition to verify inverses) MGSE9-12.F.BF.4c (Values of inverse function from graph or table)	MGSE9-12.N.CN.9 (Fundamental Theorem of Algebra) MGSE9-12.A.SSE.1,a,b (Interpret expressions; Interpret parts & terms of expressions) MGSE9-12.A.SSE.2 (Equivalent expressions) MGSE9-12.A.APR.2 (Remainder Theorem) MGSE9-12.A.APR.3 (Identify zeros) MGSE9-12.A.APR.4 (Polynomial Identities) MGSE9-12.F.IF.4 (Characteristics of functions) MGSE9-12.F.IF.7 (Graph functions) MGSE9-12.F.IF.7c (Graph polynomial functions) Cobb Honors Standards Step & Piecewise Functions Absolute Value Eq./Inequalities Inverses of Functions Infinite Geometric Series Sums of Finite Series	MGSE9-12.A.APR.7 (Rewrite rational expressions) MGSE9-12.A.CED.1 (Create equations & inequalities-1 variable) MGSE9-12.A.CED.2 (create equations & inequalities-2 variables) MGSE9-12.A.REI.2 (Solve simple radical & rational equations) MGSE9-12.F.IF.4 (Characteristics of functions) MGSE9-12.F.IF.5 (Domains of functions) MGSE9-12.F.IF.7 (Graph Functions) MGSE9-12.F.IF.7b (Graph square rt, cube rt, piecewise, step & absolute value functions) MGSE9-12.F.IF.7d (Graph rational functions)	MGSE9-12.A.SSE.3 (Equivalent expressions) MGSE9-12.A.SSE.3c (Properties of exponents) MGSE9-12.F.IF.7 (Graph functions) MGSE9-12.F.IF.7e (Graph exponential & logarithmic functions) MGSE9-12.F.IF.8 (Write a function) MGSE9-12.F.IF.8b (Interpret expressions) MGSE9-12.F.BF.5 (Inverse relationships) MGSE9-12.F.LE.4 (Express exponential models as logarithmic)	MGSE9-12.A.SSE.4 (Derive formula for sum of finite geometric series) MGSE9-12.A.CED.1 (Create equations & inequalities-1 variable) MGSE9-12.A.CED.2 (create equations & inequalities-2 variables) MGSE9-12.A.CED.3 (Represent constraints) MGSE9-12.A.CED.4 (Rearrange formulas) MGSE9-12.A.REI.11 (Solutions to equations) MGSE9-12.F.IF.6 (Average rate of change) MGSE9-12.F.IF.9 (Compare 2 functions) MGSE9-12.F.BF.3 (Build new functions from existing functions)	MGSE9-12.S.ID.2 (Shape & data distribution) MGSE9-12.S.ID.4 (Fit to a normal distribution) MGSE9-12.S.IC.1 (Inferences from a random sample) MGSE9-12.S.IC.2 (Using simulations) MGSE9-12.S.IC.3 (Randomization) MGSE9-12.S.IC.3 (Romdomization) MGSE9-12.S.IC.4 (Population mean) MGSE9-12.S.IC.5 (Compare 2 treatments) MGSE9-12.S.IC.6 (Evaluate reports based on data) Cobb Honors Standards Confidence Intervals Review: All standards by differentiating for student needs Extend: MGSE9-2.N.CN.4 (Complex plane)	
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## Algebra 2 Unit 1: Quadratics Revisited

## References

## Dear Parents,

Textbook Connection: HMH Georgia Analytic Geometry Text: Unit 5

Every student will receive a text copy and access to the online textbook resource:

### http://my.hrw.com/

### Helpful Links:

#### • Lesson on Complex Numbers: http://www.purplemath.c

om/modules/complex.htm

## • Lesson on Computing Polynomials:

http://www.purplemath.c om/modules/polymult.ht m

## • Lesson on Rational Exponents:

http://www.themathpage. com/Alg/rationalexponents.htm

## • Lesson on Operations on Complex Numbers:

https://www.khanacadem y.org/math/algebra2/intro duction-to-complexnumbers-algebra-2/thecomplex-numbersalgebra-2/v/complexnumber-intro In this unit students will:

- Define rational exponents
  - Rewrite expression involving radicals and rational exponents
- Define the imaginary number i
- Define complex numbers
- Operate with complex numbers
- Understand that the basic properties of numbers continue to hold with expressions involving exponents.

## Concepts Students will Use & Understand

- Extend the properties of exponents to rational exponents.
- Rewrite expressions involving radicals & rational exponents.
- Use properties of rational & irrational numbers to find the sum and product.
- Perform arithmetic operations with complex numbers.
- Find the conjugate of a complex number to find quotients of complex numbers

## Vocabulary

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• **Complex number**: A complex number is the sum of a real number and an imaginary number (a number whose square is a real number less than zero), i.e. an expression of the form a + bi, where *a* and *b* are real

numbers and *i* is the *imaginary unit*, satisfying  $i^2 = -1$ .

- **Exponential functions:** A function of the form  $y = a \cdot b^x$  where a > 0 and either 0 < b < 1 or b > 1.
- **<u>Nth roots</u>**: The number that must be multiplied by itself *n* times to equal a given value. The *nth* root can be notated with radicals and indices or with rational exponents, i.e.  $x^{1/3}$  means the cube root of *x*.
  - **Polynomial function** A *polynomial function* is defined as a function,  $f(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots + a_{n-2} x^2 + a_{n-3} x^1 + a_n$ , where the coefficients are real numbers.
- **<u>Rational exponents</u>**: For a > 0, and integers m and n, with n > 0,

$$a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m; a^{m/n} = (a^{1/n})^m = (a^m)^{1/n}$$

- **<u>Rational expression</u>**: A quotient of two polynomials with a non-zero denominator.
- <u>Rational number</u>: A number expressible in the form *a/b* or *a/b* for some fraction *a/b*. The rational numbers include the integers.
- For further help: http://intermath.coe.uga.edu/dictnary/homepg.asp http://www.amathsdictionaryforkids.com/

## Example 1

Combine like terms: (2+3i)+(7+i)

Answer: 9+4i

## Example 2

Find the conjugate & calculate the quotient

t: 
$$\frac{(2+5i)}{(5+2i)}$$

20 21 Answer: 29 29

## Example 3

Find the y-coordinate for the following:  $y = 2(x - 4)^2 - 5$  for x = 2.

Answer: y = 3

## Example 4

Give the value of the discriminant of the equation  $4x^2 - 8x = -4$ <mark>A) 0</mark> B) 1 C) -128

D) 128

## Example 5

Graph. Identify zeros, intervals of increase, intervals of decrease, vertex and axis of symmetry.







## Algebra II Unit 2: Operations with Polynomials

### References

### **Textbook:**

• HMH Advanced Algebra, Unit 2

### **Online Access:**

http://www.my.hrw.com

### **Helpful Links:**

- Khan Academy:
  - https://www.khanac ademy.org/math/alg ebra2/polynomial\_an d\_rational/polynomi al\_tutorial/v/termscoefficients-andexponents-in-apolynomial
- Binomial Theorem: https://www.khanac ademy.org/math/alg ebra2/polynomial\_an d\_rational/binomial\_ theorem/v/binomialtheorem

### Dividing Polynomials: https://www.khanac ademy.org/math/alg ebra2/polynomial\_an d\_rational/dividing\_p olynomials/v/polyno mial-division

### Synthetic Division:

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https://www.khanac ademy.org/math/alg ebra2/polynomial\_an d\_rational/syntheticdivision/v/syntheticdivision

### Dear Parents,

This unit develops the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students will find inverse functions and verify by composition that one function is the inverse of another function.

### **Concepts Students will Use & Understand**

- understand the definition of a polynomial
- interpret the structure and parts of a polynomial expression including terms, factors, and coefficients
- simplify polynomial expressions by performing operations, applying the distributive property, and combining like terms
- use the structure of polynomials to identify ways to rewrite them and write polynomials in equivalent forms to solve problems
- perform arithmetic operations on polynomials and understand how closure applies under addition, subtraction, and multiplication
- divide one polynomial by another using long division
- use Pascal's Triangle to determine coefficients of binomial expansion
- use polynomial identities to solve problems
- use complex numbers in polynomial identities and equations
- find inverses of simple functions

### Vocabulary

- **Coefficient:** a number multiplied by a variable.
- Degree: the greatest exponent of its variable
- End Behavior: the value of f(x) as x approaches positive and negative infinity
- **Pascal's Triangle:** an arrangement of the values of  ${}_{n}C_{r}$  in a triangular pattern where each row corresponds to a value of n
- **Polynomial:** a mathematical expression involving a sum of nonnegative integer powers in one or more variables multiplied by coefficients. A polynomial in one variable with constant coefficients can be written in

 $a_n x^n + a_{n-1} x^{n-1} + \dots a_2 x^2 + a_1 x + a_0$  form.

- **Remainder Theorem:** states that the remainder of a polynomial f(x) divided by a linear divisor (x c) is equal to f(c).
- Synthetic Division: Synthetic division is a shortcut method for dividing a
  polynomial by a linear factor of the form (x a). It can be used in place of the
  standard long division algorithm.
- **Roots**: solutions to polynomial equations.
- **Zero**: If f(x) is a polynomial function, then the values of x for which f(x) = 0 are called the **zeros** of the function. Graphically, these are the x intercepts.

### Sample Problems

Find the following products. Be sure to simplify results.

a. $3x(2x^2+8x+9)$	b. $-2x^2(5x^2-x-4)$
$6x^3 + 24x^2 + 27x$	$-10x^4 + 2x^3 + 8x^2$
c. $(2x+7)(2x-5)$	d. $(4x-7)(3x-2)$
$4x^2+4x-35$	$12x^2 - 29x + 14$
e. $(x-3)(2x^2+3x-1)$	f. $(6x+4)(x^2-3x+2)$
$2x^3 - 3x^2 - 10x + 3$	$6x^3 - 14x^2 + 8$
g. $(4x-7y)(4x+7y)$	h. $(3x-4)^2$
$16x^2 - 49y^2$	$9x^2 - 24x + 16$
i. $(x-1)^3$	j. $(x-1)^4$
$x^3 - 3x^2 + 3x - 1$	$x^4 - 4x^3 + 6x^2 - 4x - 1$

Description	Identity
Difference of Two Squares	$(a+b)(a-b) = a^2 - b^2$
Sum of Two Squares	$(a+bi)(a-bi) = \mathbf{a^2} + \mathbf{b^2}$
Perfect Square Trinomial	$(a+b)^2 = a^2 + 2ab + b^2$
Perfect Square Trinomial	$(a-b)^2 = a^2 - 2ab + b^2$
Binomial Cubed	$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
Binomial Cubed	$(a-b)^3 = a^3 - 3a^2b + 3ab^2 + b^3$
Sum of Two Cubes	$a^{3} + b^{3} = (a + b) (a^{2} - ab + b^{2})$
Difference of Two Cubes	$a^{3}-b^{3} = (a - b)(a^{2} + ab + b^{2})$

Divide:  $x - 2 \sqrt{x^3 + 2x^2 - 5x - 6}$ Long Division vs. Synthetic Division

$$\begin{array}{r} 4x^3 + 5x^2 + 3x + 2 \\ \underline{x-2} 4x^4 - 3x^3 - 7x^2 - 4x - 9 \\ \underline{4x^4 - 8x^3} \\ 5x^3 - 7x^2 \\ \underline{5x^3 - 10x^2} \\ 3x^2 - 4x \\ \underline{3x^2 - 6x} \\ \underline{2x - 9} \\ \underline{2x - 4} \\ -5 \end{array}$$

$$\begin{array}{r} 4 & 5 & 3 & 2 \\ \underline{-2} 4 - 3 - 7 - 4 - 9 \\ \underline{-3} \\ 5 \\ \underline{-10} \\ 3 \\ \underline{-10}$$



## Algebra II Unit 3: Polynomials Functions

### References

### Textbook:

 HMH Advanced Algebra, Unit 2 Module 4

### **Online Access:**

http://www.my.hrw.com

### **Helpful Links:**

- GA Virtual Learning http://cms.gavirtualsch ool.org/Shared/Math/G SEAdvancedAlgebra/P olynomialFunctions/in dex.html
- Fundamental Theorem of Algebra
   https://mathbitsnoteb
   ook.com/Algebra2/Qua
   dratics/QDfundamenta
   IThm html
- The Polynomial Remainder Theorem https://mathbitsnoteb ook.com/Algebra2/Pol ynomials/PORemainde rTh.html
- Polynomial Identities https://mathbitsnoteb ook.com/Algebra2/Pol ynomials/POIdentity.ht ml
- Graph Polynomial Functions

https://mathbitsnoteb ook.com/Algebra2/Pol ynomials/POGraphing. html

 MathBitsNotebook Algebra 2

> http://mathbitsnotebo ok.com/Algebra2/Alge bra2.html

### Dear Parents,

In this unit, students continue their study of polynomials by identifying zeros and making connections between zeros of a polynomial and solutions of a polynomial equation. Students will see how the Fundamental Theorem of Algebra can be used to determine the number of solutions of a polynomial equation and will find all the roots of those equations. Students will graph polynomial functions and interpret the key characteristics of the function

### **Concepts Students will Use & Understand**

- use polynomial identities to solve problems
- use complex numbers in polynomial identities and equations
- understand and apply the rational Root Theorem
- understand and apply the Remainder Theorem
- understand and apply The Fundamental Theorem of Algebra
- understand the relationship between zeros and factors of polynomials
- represent, analyze, and solve polynomial functions algebraically and graphically

### Vocabulary

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**End Behavior:** the value of f(x) as x approaches positive and negative infinity **Relative Minimum:** a point on the graph where the function is increasing as you move away from the point in the positive and negative direction along the horizontal axis. **Relative Maximum:** a point on the graph where the function is decreasing as you move away from the point in the positive and negative direction along the horizontal axis. **Fundamental Theorem of Algebra:** every non-zero single-variable polynomial with complex coefficients has exactly as many complex roots as its degree, if each root is counted up to its multiplicity.

**Multiplicity**: the number of times a root occurs at a given point of a polynomial equation.

**Pascal's Triangle:** an arrangement of the values of  ${}_{n}C_{r}$  in a triangular pattern where each row corresponds to a value of n

**Rational Root Theorem:** a theorem that provides a complete list of all possible rational roots of a polynomial equation. It states that every rational zero of the polynomial equation  $f(x) = a_n x^n + a_{n-1} x^{n-1} + ... a_2 x^2 + a_1 x + a_0$ , where all coefficients are integers, has the

following form:  $\frac{p}{q} = \frac{\text{factors of constant term } a_0}{\text{factors of leading coefficient } a_n}$ 

**Remainder Theorem:** states that the remainder of a polynomial f(x) divided by a linear divisor (x - c) is equal to f(c)

### **Sample Problems**

1. The height of an arrow shot by a 6 foot tall person is given by the function equation image indicator where h is the height and t is the time. At what time would the arrow be able to hit a target 10 feet in the air?

The arrow could hit a 10 foot target in 2 sec. or in 2 2/3 sec.

2. Draw a rough sketch of the graph of  $y = -x^2 + 4x - 3$ 



3. A soccer ball is kicked from the ground. The height of the ball is modeled by the equation  $h(t) = -4.9t^2 + 19.6t$ Height is in meters. Time is in seconds. How long is the ball in the air?

### <mark>4 seconds</mark>

4. Describe the key features of the following polynomial function:  $f(x) = x^4 + x^2 - 20$ 



End behavior:

 $x \to -\infty, f(x) \to \infty; \ x \to \infty, f(x) \to \infty$ 



## Algebra II Unit 4: Radical & Rational Relationships

### References

### Textbook:

• HMH Advanced Algebra, Unit 3 Modules 6-7

### **Online Access:**

http://www.my.hrw.com

### **Helpful Links:**

- GA Virtual Learning http://cms.gavirtuals chool.org/Shared/Ma th/GSEAdvancedAlge bra/RationalAndRadi calRelationships/inde x.html
- Khan Academy https://www.khanac ademy.org/math/alg ebra2/rationalexpressionsequations-andfunctions
  - Radical Functions http://www.purplem ath.com/modules/gr aphrad.htm
  - Rational Functions:

http://www.purplem ath.com/modules/gr phrtnl.htm

### **Dear Parents,**

Students investigate two more types of families of functions – radical and rational functions. They also learn how to simplify expressions and solve equations and inequalities that involve rational and radical expressions. Below you find will several resources that can be used to help support the learning for your child.

### **Concepts Students will Use & Understand**

- Explore Rational and Radical Functions
- Determine rational numbers extend the arithmetic of integers by allowing division by all numbers except zero. Similarly, rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial
- Notice the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers
- Investigate the properties of simple rational and radical functions and then expand their knowledge of the graphical behavior and characteristics of more complex rational functions
- Recall and make use of their knowledge of polynomial functions as well as compositions of functions to investigate the characteristics of these more complex rational functions
- Solve equations and inequalities involving rational and radical functions
- Understand that not all solutions generated algebraically are actually solutions to the equations and extraneous solutions will be explored
- Apply these rational and radical functions with an emphasis on interpretation of real world phenomena as it relates to certain characteristics of the rational expressions

### Vocabulary

- **Radical Function:** A function containing a root. The most common radical functions are the square root and cube root functions:  $f(x) = \sqrt{x}$  and  $g(x) = \sqrt[3]{x}$ .
- **Rational Function:** The quotient of two polynomials, P(z) and Q(z), where  $R(z) = \frac{P(z)}{Q(z)}$

### **Sample Problems**

Simplify each expression or solve each equation or inequality.



3a.  $\sqrt{x+4} = 3\sqrt{x}$ 

3b. 
$$4(x-12)^{\frac{1}{3}} = -16$$

3c. 
$$\sqrt{-14x+2} = x-3$$

4. Graph: 
$$f(x) = \sqrt[3]{x} + 1$$
. State Domain and Range.



- f(x) is a radical function whose domain is (-∞, 5] and whose range is (-∞, 0], and g(x) = √(x+5)+2. Which statement is true for both f(x) and g(x)?
  - A. g(x) has a higher y-intercept than f(x)
  - B. g(x) has a greater zero than f(x)
  - C. The functions have the same domain.
  - D. The functions have the same range.



## Algebra II Unit 5: Exponential & Logarithmic Functions

## References

Textbook Connection: HMH Georgia Advanced Algebra Text: Unit 4: Modules 8 & 9

Every student will receive a text copy and access to the online textbook resource:

### http://my.hrw.com/

### Helpful Links:

### □ GA Virtual:

http://cms.gavirtualsc hool.org/Shared/Math /GSEAdvancedAlgebra /ExponentialAndLogar ithms/index.html

### □ Regents Prep:

http://regentsprep.or g/regents/math/algtri g/atp8b/logFunction.h tm

### □ Khan Academy:

https://www.khanaca demy.org/math/algeb ra2/exponential\_and\_ logarithmic\_func

### Purple Math:

http://www.purplema th.com/modules/grap hlog.htm

### □ The Math Page:

http://www.themathp age.com/aprecalc/log arithmic-exponentialfunctions.htm

## Dear Parents,

Students extend their work with exponential functions to include solving exponential equations with logarithms. They analyze the relationship between these two functions. In this unit students will:

- Review exponential functions and their graphs
- Explore exponential growth
- Develop the concept of a logarithm as an exponent along with the inverse relationship with exponents
- Define logarithms and natural logarithms
- Develop the change of base formula
- Develop the concept of logarithmic function
- Solving problems relating to exponential functions and logarithms

## **Concepts Students will Use & Understand**

- The concept of a function
- Various representations of functions
- Exponential functions and characteristics of their graphs
- The solution of linear equations using algebra and graphing approaches
- Familiarity with graphing technology
- Use patterns to write a function to model a situation

## Vocabulary

- Asymptote: An asymptote is a line or curve that approaches a given curve arbitrarily closely. A graph never crosses a vertical asymptote, but it may cross a horizontal or oblique asymptote.
- **Common logarithm:** A logarithm with a base of 10. A common logarithm is the exponent, *a*, such that  $10^a = b$ . The common logarithm of *x* is written *log x*. For example, *log 100* = 2 because  $10^2 = 100$ .
- **Continuously compounded interest:** Interest that is, theoretically, computed and added to the balance of an account each instant. The formula is  $A = Pe^{rt}$ , where A is the ending amount, P is the principal or initial amount, r is the annual interest rate, and t is the time in years.
- **Compounded interest:** A method of computing the interest, after a specified time, and adding the interest to the balance of the account. Interest can be computed as little as

once a year to as many times as one would like. The formula is  $A = P(1 + \frac{r}{n})^{nt}$  where A is the ending amount, P is the principal or initial amount, r is the annual interest rate, n is the number of times compounded per year, and t is the number of years.

- **Exponential functions:** A function of the form  $y = a^x$  where a > 0 and  $a \neq 1$ .
- **Logarithmic functions:** A function of the form  $y = \log_b x$  with  $b \neq 1$  and b and x both positive. A logarithmic function is the inverse of an exponential function. The inverse of  $y = b^x$  is  $y = \log_b x$ .

- **Logarithm:** The logarithm base *b* of a number x,  $\log_b x$ , is the exponent to which *b* must be raised to equal x.
- Natural exponential: Exponential expressions or functions with a base of e; i.e.,  $y = e^{x}$ .
- Natural logarithm: A logarithm with a base of *e*. ln b is the exponent, *a*, such that  $e^a = b$ . The natural logarithm of *x* is written ln x and represents  $log_e x$ . For example, ln 8 = 2.0794415... because  $e^{2.0794415...} = 8$ .

For further help:

http://www.teachers.ash.org.au/jeather/maths/dictionary.html http://intermath.coe.uga.edu/dictnary/homepg.asp http://www.amathsdictionaryforkids.com/

### **Sample Practice Problems**

1) State the domain and range for  $f(x) = -2^x + 4$ 

D: all real numbers; R: y < 4

2) State the domain and range for  $3\log_5 x$ 

D: x > 0; R: all real numbers

3) Solve  $2(3)^{2x} = 5$ x = 0.417

4) Solve  $5\log(x-2) = 11$ x = 160.49

5) Identify asymptotes, y-intercept and point of maximum growth:  $y = \frac{2}{1 + e^{-2x}}$ 

Asymptotes: x-axis and y=2; y-intercept (0,1); maximum growth is at (0,1)

6) Find the inverse of the function  $y = \log_6 x$ 

y = 6x

7) The value of a new car purchased for \$24,900 decreases by 10% per year. Write an exponential decay model for the value of the car. After about how many years will the car be worth half its purchase price?

V(t) = 24,9000(0.90)t; about 6.58 years

8) You deposit \$4,000 in an account that pays 7% annual interest compounded continuously. Find the balance at the end of five years.

\$5,676.27



## Algebra II **Unit 6: Mathematical Modeling**

## References

## **Dear Parents**,

In this unit students will:

- Synthesize and generalize what they have learned about a variety of function families •
- derive the formula for the sum of a finite geometric series and use it to solve problems •
- Explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying functions
- Identify appropriate types of functions to model a situation, •
- Adjust parameters to improve the model, •
- Compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit
- Determine whether it is best to model with multiple functions creating a piecewise function

## **Concepts Students will Use & Understand**

- quantitative reasoning
- solving various functions (finding zeros) through factoring, using other algebraic processes, using geometry, or by graphing
- properties of exponents and the associated properties of logarithms •
- a working knowledge of geometric vocabulary •
- writing explicit and recursive formulas for geometric sequences •
  - the ability to recall and apply basic algebraic and geometric processes
- an ability to understand mathematics through a variety of representations
- familiarity with technology, particularly the graphing calculator •
- prior knowledge and understanding of functions learned earlier in the course, as this is the culminating unit

### □ Math Bits Notebook:

https://mathbitsnoteb ook.com/Algebra2/Se quences/SSGeometric. html

□ Math Bits Notebook: http://mathbitsnoteb ook.com/Algebra1/Fu nctionGraphs/FNGTyp ePiecewise.html

### □ Wolfram Math World: http://mathworld.wol fram.com/GeometricS

### Vocabulary

•

- Geometric Sequence: is a sequence with a constant ratio between successive terms
- Geometric Series: the expression formed by adding the terms of a geometric sequence
- **Recursive:** A type of sequence in which successive terms are generated by preceding • terms in the sequence.
- Sum of a finite geometric series: The sum, S<sub>n</sub>, of the first *n* terms of a geometric sequence is given by  $S_n = \frac{a_1 - a_1 r^n}{1 - r} = \frac{a_1 (1 - r^n)}{1 - r}$ , where  $a_1$  is the first term and r is the common ratio ( $r \neq 1$ ).
- Sum of an infinite geometric series: The general formula for the sum S of an infinite

geometric series  $a_1 + a_2 + a_3 + \dots$  with common ratio r where |r| < 1 is  $S = \frac{a_1}{1 - r}$ . If an

### **Textbook Connection: HMH Advanced Algebra Text:** Unit 2: Module 5 & Unit 6: Modules 12-15

Every student will receive a text copy and access to the online textbook resource: http://my.hrw.com/

### **Helpful Links:**

### □ GA Virtual:

http://cms.gavirtualsc hool.org/Shared/Math /GSEAdvancedAlgebra /MathematicalModeli ng/index.html

### □ Khan Academy:

https://www.khanaca demy.org/math/algeb ra2/functions-andgraphs/piecewisefunctionstutorial/v/graphs-ofabsolute-valuefunctions

#### eries.html

#### □ Purple Math:

http://www.purplema th.com/modules/serie s5.htm infinite geometric series has a sum, i.e. if |r| < 1, then the series is called a **convergent** geometric series. All other geometric (and arithmetic) series are **divergent**.

### For further help:

http://intermath.coe.uga.edu/dictnary/homepg.asp http://www.amathsdictionaryforkids.com/

### Sample Practice Problems

1. The price of dance lesson depends upon the number of lessons that you select. If *x* is the number of lessons then the fee for the lessons (in dollars) can be found using the piecewise function

 $f(x) = \begin{cases} 40x \ if \ 0 < x \le 4\\ 30x \ if \ 4 < x \le 8\\ 25x \ if \ x > 8 \end{cases}$ 

The lessons are increasing by 10% per lesson with a \$5 processing fee for each student. What is the new function for the cost of lessons?

 $f(x) = \begin{cases} 44x + 5 & \text{if } 0 < x \le 4\\ 33x + 5 & \text{if } 4 < x \le 8\\ 27.5x + 5 & \text{if } x > 8 \end{cases}$ 

2. Use the function  $f(x)\sqrt[3]{5x}$  to answer the following questions: What is the domain & range? Domain: all real numbers; Range: all real numbers

What is the inverse of f(x)?

$$f^{-1}(x) = \frac{\pi}{5}$$

What is the domain and range of the inverse function? Domain: all real numbers; Range: all real numbers

Over what line does the function and its inverse reflect across on the coordinate plane? y = x

Identify the axis of symmetry, vertex, intercepts, domain, range, slope, and max/min of the following absolute value function:
 f(x) = |x| + 3

A.O.S: x = 0; Vertex: (0, 3); x-intercept: none; y-intercept: 3; Domain: all real numbers; Range:  $y \ge 3$ ; Left Slope: - 1; Right Slope: 1; Minimum: 3



## Algebra II Unit 7: Inferences & Conclusions from Data

## References

Textbook Connection: HMH Advanced Algebra Text: Unit 1: Module 1-2

Every student will receive a text copy and access to the online textbook resource: http://my.hrw.com/

### Helpful Links:

Measures of Central Tendency - video Box and Whisker Plots video Variance and Standard Deviation - video Normal Distribution video Types of Statistical Studies - video

Comparing and Sampling Populations- video Sampling Distribution video Confidence Intervals video Margin of Error - video Hypothesis Testing video

## Dear Parents,

In this unit, students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data— including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn.

## **Concepts Students will Use & Understand**

- Construct appropriate graphical displays (dot plots, histogram, and box plot) to represent sets of data values.
- Describe a distribution using shape, center and spread and use the correct measure appropriate to the distribution.
- Compare two or more different data sets using center and spread.
- Recognize data that is described well by a normal distribution.
- Estimate probabilities for normal distributions using area under the normal curve using calculators, spreadsheets and tables.
- Design a method to select a sample that represents a variable of interest from a population.
- Design simulations of random sampling and explain the outcomes in context of population and known proportions or means.
- Use sample means and proportions to estimate population values and calculate margins of error.
- Read and explain in context data from real-world reports.

### Vocabulary

- Center: measures of center such as median & mean.
- **Central Limit Theorem:** the mean of a sample of data will be closer to the mean of the overall population as the sample size increases.
- **Confidence Interval**: the probability that the interval reveals the true parameter value.
- Empirical Rule: in a normal distribution, the data will fall into 3 standards deviations of the mean.
- **Margin of Error:** the value in the confidence interval that shares how accurate the parameter may be.
- **Parameters:** numerical values that describe a population.
- Sample Mean: a statistic measuring the average of the observation in the sample.
- Sample Proportion: a statistic indicating the proportion of successes in a particular sample.
- **Sampling Distribution:** the distribution of values taken by a statistic in all possible same size and population samples.
- Sampling Variability: the fact that the value of a statistic varies in repeated random sampling.
- **Shape:** distribution described by symmetry, number of peaks, direction of skew, or uniformity.
- **Spread:** variability of the data.
- Standard Deviation: the square root of the variance.
- Variance: the average of the squares of the deviations of the observations from their mean.

## **Sample Practice Problems**

### Data Collection and Design

1. Adam rolled a six sided die 4 times and obtained the following results: 5, 5, 3, and 4. He computed the mean of the 4 rolls and used the result to estimate the mean of the population. Identify the parameter, sample, and statistic of interest. Calculate the identified statistic.

2. The Bennett family believes that they have a special genetic makeup because there are 5 children in the family and all of them are girls. Create a simulation of 100 families with 5 children using coins. Determine the percent of families in which all 5 children are girls and interpret your results.

3. Your classmate Jimmy presents a project to your class about carcinogens, substances that can cause cancer in living cells. When Jimmy said during his presentation that some soda ingredients may be carcinogens, you nearly spit out your root beer. Now you can't rest until you know whether soda consumption is linked to developing cancer. How would you go about investigating whether soda and cancer are linked? What are the benefits and drawbacks of using surveys, experiments, or observational studies?

### Summarizing and Interpreting Data

1. Two rival basketball teams each have ten players on a team. The total points scored by each player in the first five games of the season are shown below.

Cougars: 21, 30, 8, 41, 11, 21, 26, 28, 32, 30 The coaches want to compare the points scored by a typical player on each team. What statistics should the coaches use? Create a box plot and compare those statistics. Then compare any other statistics that are appropriate so that center and spread are compared for both data sets. Identify any outliers and explain their effects.

2. Two small start-up companies are hiring. Josefina, who is interviewing for jobs at both companies, is comparing the salaries of the companies' current employees. The representative for Company A says her company's typical salary is \$42,000 per year. The company B representative says his company's typical salary is \$63,000 per year. The actual salaries, in thousands of dollars are shown below.

Company A: 21, 33, 35, 40, 42, 45, 45, 49, 160 Do the figures given by the company representatives really represent the typical salaries for each company? Based on the current employee's salaries, at which company is Josefina likely to earn more money. Explain.

### Normal Distribution Curve and its Applications

1. If a population of human body temperatures is normally distributed with a mean of 98.2 ° F and a standard deviation of 0.7 ° F, estimate the percent of temperatures between 98.0 ° F and 99.0 ° F.

### Conclusions from Data

1. In 2011, the average salary for a sample of NCAA Division 1 head football coaches was \$1.5 million per year, with a standard deviation of \$1.07 million. If there are 100 coaches in this sample, what is the standard error of the mean? What can your predict about the population mean based on the sample mean and its standard error?

2. A group of marine biologists placed tracking tags on 100 fish in Lake Erie one summer. The weight of each fish was recorded at the beginning and end of the summer. The average weight gain for all of the tagged fish was 1.2 pounds, with a standard deviation of 0.4 pound. What is the margin of error with 90% confidence for this study?



Geometry Teaching & Learning Framework						
Block Schedule						
Unit 1 1.5 weeks	Unit 2 5.5 weeks	Unit 3 2 weeks	Unit 4 3.5 weeks	Unit 5 2.5 weeks	Unit 6 3 weeks	
Transforma- tions in the Coordinate Plane	Similarity, Congruence & Proofs	Right Triangle Trigonometry	Circles & Volume	Geometric & Algebraic Connections	Applications of Probability Review & Extend	
MGSE9- 12.G.CO.1 (Precise definitions) MGSE9- 12.G.CO.2 (Coordinate plane) MGSE9- 12.G.CO.3 (Figures with rotations & reflections upon itself) MGSE9- 12.G.CO.4 (Definitions of transformations) MGSE9- 12.G.CO.5 (Transforming figures)	MGSE9-12.G.SRT.1-2 (Dilations & similarity) MGSE9-12.G.SRT.3 (AA criterion) MGSE9-12.G.SRT.4 (Prove theorems about triangles) MGSE9-12.G.SRT.5 (Congruence & similarity) MGSE9-12.G.CO.6-7 (Congruence & rigid motions) MGSE9-12.G.CO.8 (Triangle congruence) MGSE9-12.G.CO.9-11 (Prove geometric theorems) MGSE9-12.G.CO.12 (Geometric constructions) MGSE9-12.G.CO.13 (Construct regular polygons inscribed in a circle)	MGSE9-12.G.SRT.6 (Trigonometric ratios) MGSE9-12.G.SRT.7 (Sine & cosine of complementary angles) MGSE9-12.G.SRT.8 (Trigonometric ratios & Pythagorean Theorem)	MGSE9-12.G.C.1-2 (Similar circles; radii, chords, tangents & secants with inscribed, central & circumscribed angles) MGSE9-12.G.C.3-5 (Constructing inscribed & circumscribed circles; construct a tangent line; derive arc lengths) MGSE9-12.G.GMD.1 (Informal arguments for geometric formulas) MGSE9-12.G.GMD.2-4 (Cavalieri's principle; volume; cross-sections & rotations)	MGSE9-12.G.MG.1-3 (Describe objects; density; design problems) MGSE9-12.G.GPE.1 (Derive the equation of a circle) MGSE9-12.G.GPE.4 (Coordinates to prove simple geometric theorems) MGSE9-12.G.GPE.5-7 (Prove the slope criteria; partition a line segment; compute perimeters using the distance formula)	MGSE9-12.S.CP.1-4 (Set theory; independent probability; two-way tables) MGSE9-12.S.CP.5 (Recognize & explain conditional probability) MGSE9-12.S.CP.6-7 (Probability of compound events) <u>Review</u> : All standards by differentiating for student needs <u>Extend</u> : MGSE9-12.N.CN.1 (Complex numbers)	
These units were written to build upon concepts from prior units, so later units contain tasks that depend upon the concepts addressed in earlier units. All units will include the Mathematical Practices and indicate skills to maintain.						

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Grades 9-12 Key: Algebra Strand: SSE = Seeing Structure in Expressions, APR = Arithmetic with Polynomial and Rational Expressions, CED = Creating Equations, REI = Reasoning with Equations and Inequalities Functions Strand: IF = Interpreting Functions, LE = Linear and Exponential Models, BF = Building Functions, TF = Trigonometric Functions

Geometry Strand: CO = Congruence, SRT = Similarity, Right Triangles, and Trigonometry, C = Circles, GPE = Expressing Geometric Properties with Equations, GMD = Geometric Measurement and Dimension, MG = Modeling with Geometry



Geometry Teaching & Learning Framework							
Semester 1			Semester 2				
Unit 1 3 weeks Transforma- tions in the Coordinate Plane	Unit 2 11 weeks Similarity, Congruence & Proofs	Unit 3 4 weeks Right Triangle Trigonometry	Unit 4 7 weeks Circles & Volume	Unit 5 <sup>5 weeks</sup> Geometric & Algebraic Connections	Unit 6 <sup>6 weeks</sup> Applications of Probability		
MGSE9- 12.G.CO.1 (Precise definitions) MGSE9- 12.G.CO.2 (Coordinate plane) MGSE9- 12.G.CO.3 (Figures with rotations & reflections upon itself) MGSE9- 12.G.CO.4 (Definitions of transformations) MGSE9- 12.G.CO.5 (Transforming figures)	MGSE9-12.G.SRT.1-2 (Dilations & similarity) MGSE9-12.G.SRT.3 (AA criterion) MGSE9-12.G.SRT.4 (Prove theorems about triangles) MGSE9-12.G.SRT.5 (Congruence & similarity) MGSE9-12.G.CO.6-7 (Congruence & rigid motions) MGSE9-12.G.CO.8 (Triangle congruence) MGSE9-12.G.CO.9-11 (Prove geometric theorems) MGSE9-12.G.CO.12 (Geometric constructions) MGSE9-12.G.CO.13 (Construct regular polygons inscribed in a circle)	MGSE9-12.G.SRT.6 (Trigonometric ratios) MGSE9-12.G.SRT.7 (Sine & cosine of complementary angles) MGSE9-12.G.SRT.8 (Trigonometric ratios & Pythagorean Theorem)	MGSE9-12.G.C.1-2 (Similar circles; radii, chords, tangents & secants with inscribed, central & circumscribed angles) MGSE9-12.G.C.3-5 (Constructing inscribed & circumscribed circles; construct a tangent line; derive arc lengths) MGSE9-12.G.GMD.1 (Informal arguments for geometric formulas) MGSE9-12.G.GMD.2-4 (Cavalieri's principle; volume; cross-sections & rotations)	MGSE9-12.G.MG.1-3 (Describe objects; density; design problems) MGSE9-12.G.GPE.1 (Derive the equation of a circle) MGSE9-12.G.GPE.4 (Coordinates to prove simple geometric theorems) MGSE9-12.G.GPE.5-7 (Prove the slope criteria; partition a line segment; compute perimeters using the distance formula)	Review & Extend MGSE9-12.S.CP.1-4 (Set theory; independent probability; conditional probability; two-way tables) MGSE9-12.S.CP.5 (Recognize & explain conditional probability) MGSE9-12.S.CP.6-7 (Probability of compound events) <u>Review</u> : All standards by differentiating for student needs <u>Extend</u> : MGSE9-12.N.CN.1 (Complex numbers)		
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Honors Geometry Teaching & Learning Framework						
Semester 1			Semester 2			
Unit 1 3 weeks	Unit 2 11 weeks	Unit 3 4 weeks	<b>Unit 4</b> 7 weeks	<b>Unit 5</b> 5 weeks	Unit 6 6 weeks	
Transforma- tions in the Coordinate Plane	Similarity, Congruence & Proofs	Right Triangle Trigonometry	Circles & Volume	Geometric & Algebraic Connections	Applications of Probability Review & Extend	
MGSE9- 12.G.CO.1 (Precise definitions) MGSE9- 12.G.CO.2 (Coordinate plane) MGSE9- 12.G.CO.3 (Figures with rotations & reflections upon itself) MGSE9- 12.G.CO.4 (Definitions of transformations) MGSE9- 12.G.CO.5 (Transforming figures)	MGSE9-12.G.SRT.1-2 (Dilations & similarity) MGSE9-12.G.SRT.3 (AA criterion) MGSE9-12.G.SRT.4 (Prove theorems about triangles) MGSE9-12.G.SRT.5 (Congruence & similarity) MGSE9-12.G.CO.6-7 (Congruence & rigid motions) MGSE9-12.G.CO.8 (Triangle congruence) MGSE9-12.G.CO.9-11 (Prove geometric theorems) MGSE9-12.G.CO.12 (Geometric constructions) MGSE9-12.G.CO.13 (Construct regular polygons inscribed in a circle) Cobb Honors Standards Conditional Statements Special Quadrilaterals	MGSE9-12.G.SRT.6 (Trigonometric ratios) MGSE9-12.G.SRT.7 (Sine & cosine of complementary angles) MGSE9-12.G.SRT.8 (Trigonometric ratios & Pythagorean Theorem) Cobb Honors Standards Elevation & Depression	MGSE9-12.G.C.1-2 (Similar circles; radii, chords, tangents & secants with inscribed, central & circumscribed angles) MGSE9-12.G.C.3-5 (Constructing inscribed & circumscribed circles; construct a tangent line; derive arc lengths) MGSE9-12.G.GMD.1 (Informal arguments for geometric formulas) MGSE9-12.G.GMD.2-4 (Cavalieri's principle; volume; cross-sections & rotations) Cobb Honors Standards Surface Area of a Sphere	MGSE9-12.G.MG.1-3 (Describe objects; density; design problems) MGSE9-12.G.GPE.1 (Derive the equation of a circle) MGSE9-12.G.GPE.4 (Coordinates to prove simple geometric theorems) MGSE9-12.G.GPE.5-7 (Prove the slope criteria; partition a line segment; compute perimeters using the distance formula) Cobb Honors Standards Loci	MGSE9-12.S.CP.1-4 (Set theory; independent probability; conditional probability; two-way tables) MGSE9-12.S.CP.5 (Recognize & explain conditional probability) MGSE9-12.S.CP.6-7 (Probability of compound events) <u>Review</u> : All standards by differentiating for student needs <u>Extend:</u> MGSE9-12.N.CN.1 (Complex numbers)	
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## **Geometry Unit 1** Transformations in the Coordinate Plane

### References Textbook Connection

HMH Coordinate Algebra Unit 5: Modules 16-17

Online Textbook Access: http://my.hrw.com

# Ask your teacher for log in directions.

### Helpful Links:

http://www.mathwarehou se.com/transformations/

http://www.gradeamathh elp.com/transformationgeometry.html

http://www.onlinemathle arning.com/transformatio n-in-geometry.html

http://mathbitsnotebook. com/Geometry/Transform ations/TRRigidTransformat ions.html

http://cms.gavirtualschool .org/Shared/Math/GSECoo rdinateAlgebra/Transform ations/index.html

## Dear Parents

In this unit students will take a closer look at translations, rotations, and reflections on the coordinate plane. Students will develop a better understanding of transformations by using a variety of tools.

## Concepts Students will Use & Understand

- Know precise definitions of geometric figures
  - Represent transformations in the plane and describe as functions
- Describe the rotations/reflections given a rectangle, parallelogram, trapezoid or regular polygon that carry it onto itself
- Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines and line segments
- Given a geometric figure and a rotation, reflection or translation, draw the transformed figurespecify a sequence of transformations that will carry a given figure onto another.

## Vocabulary

- Angle: A figure created by two distinct rays that share a common endpoint (also known as a vertex). ∠ABC or ∠B or ∠CBA indicate the same angle with vertex B.
- Angle of Rotation: The amount of rotation (in degrees) of a figure about a fixed point such as the origin.
  - Bisector: A point, line or line segment that divides a segment or angle into two equal parts.
- Circle: The set of all points equidistant from a point in a plane.
- Congruent: Having the same size, shape and measure. ∠A ≃ ∠B indicates that angle A is congruent to angle B.
- Corresponding angles: Angles that have the same relative position in geometric figures.
- Corresponding sides: Sides that have the same relative position in geometric figures.
- Endpoint: The point at each end of a line segment or at the beginning of a ray.
- Image: The result of a transformation.
- Intersection: The point at which two or more lines intersect or cross.
- Isometry: a distance preserving map of a geometric figure to another location using a reflection, rotation or translation. M → M' indicates an isometry of the figure M to a new location M'. M and M' remain congruent.
- Line: One of the undefined terms of geometry that represents an infinite set of points with no thickness and its length continues in two opposite directions indefinitely.  $\overrightarrow{AB}$  indicates a line that passes through points A and B.
- Line segment: A part of a line between two points on the line.  $\overline{AB}$  indicates the line segment between points A and B.
- Parallel lines: Two lines are parallel if they lie in the same plane and do not intersect.  $\overrightarrow{AB} \parallel \overrightarrow{CD}$  indicates that line AB is parallel to line CD.
- **Perpendicular lines:** Two lines are perpendicular if they intersect to form right angles.  $\overrightarrow{AB} \perp \overrightarrow{CD}$  indicates that line AB is perpendicular to line CD.
- **Point:** One of the basic undefined terms of geometry that represents a location. A dot is used to symbolize it and it is thought of as having no length, width or thickness.
- **Pre-image:** A figure before a transformation has taken place.
- **Ray:** A part of a line that begins at a point and continues forever in one direction.  $\overrightarrow{AB}$  indicates a ray that begins at point A and continues in the direction of point B indefinitely.
- Reflection: A transformation of a figure that creates a mirror image, "flips," over a line.

- Reflection Line (or line of reflection): A line that acts as a mirror so that corresponding points are the same distance from the mirror.
- Rotation: A transformation that turns a figure about a fixed point through a given angle and a given direction, such as 90° clockwise.
- Segment: See line segment.
- **Transformation:** The mapping, or movement, of all points of a figure in a plane according to a common operation, such as translation, reflection or rotation.
- Translation: A transformation that slides each point of a figure the same distance in the same direction.
- Vertex: The location at which two lines, line segments or rays intersect.

Try http://intermath.coe.uga.edu/dictnary/homepg.asp or http://www.amathsdictionaryforkids.com/ for further examples.

### Example 1

```
Skill-based Task
```

Which of the following preserves distance and which does not?  $(x, y) \rightarrow (x + 1, y + 2)$  $(x, y) \rightarrow (x^2, y + 1)$ 

### Example 2

y = -x

Translation (x, y)  $\rightarrow$  (x + 4, y - 2). Rotation 180° about the origin. Reflection about the line



### Example 3

Identify the coordinates of point (-7, -6) under the rotation of 90° clockwise about the origin? a. (7, 6)

b. (6, -7) c. (-6, 7) d. (-7, 6)

### Кеу

#### Example 1

The first one preserves distance since it is a translation with adding and subtracting. The second one has a quadratic applied, so the distance is not constant.

Example 2



Black to blue to yellow to red.

Example 3

С



## <u>Geometry Unit 2</u> Similarity, Congruence, and Proofs

#### References

#### Textbook Connection:

Holt McDougal Analytic Geometry: Unit 1

#### Helpful Links:

#### □ Dilations:

http://mathbitsnote book.com/Geometr y/Transformations/T RTransformationDila tions.html

- Dilations: http://mathbitsnote book.com/Geometr y/Similarity/SMdilati on.html
- Similarity: http://mathbitsnote book.com/Geometr y/Similarity/SMSimil ar.html
- Proving Similar
   Triangles:
   http://mathbitsnote

book.com/Geometr y/Similarity/SMProo fs.html

Triangle Theorems: http://mathbitsnote book.com/Geometr y/CongruentTriangle s/CTtriangleMethod s.html

Ratio Segments: http://www.walterfendt.de/m14e/pro psegments.htm

Congruent Triangles: http://www.analyze math.com/Geometr y/congruent\_triangl es.html

 Points of Concurrency: http://www.online mathlearning.com/c

### **Dear Parents**

In this unit, students will understand similarity in terms of similarity transformations, prove theorems involving similarity, understand congruence in terms of rigid motions, prove geometric theorems, and make geometric constructions.

## Concepts Students will Use & Understand

- Understand similarity in terms of similarity transformations (dilations).
- Prove theorems involving similarity (proportionality & Pythagorean Theorem)
- Understand congruence in terms of rigid motion (ASA, SAS, SSS)
- Prove geometric theorems (special angles, triangles, parallelograms)
- Make geometric constructions ( copy segment/angle; bisect segment/angle; construct perpendicular/parallel lines; equilateral triangle, square and a regular hexagon inscribed in a circle

#### Vocabulary

- Adjacent Angles: Angles in the same plane that have a common vertex and a common side, but no common interior points.
- Alternate Exterior Angles: Alternate exterior angles are pairs of angles formed when a third line (a transversal) crosses two other lines. These angles are on opposite sides of the transversal and are outside the other two lines. When the two other lines are parallel, the alternate exterior angles are equal.
- Alternate Interior Angles: Alternate interior angles are pairs of angles formed when a third line (a transversal) crosses two other lines. These angles are on opposite sides of the transversal and are in between the other two lines. When the two other lines are parallel, the alternate interior angles are equal.
- **Bisector:** A bisector divides a segment or angle into two equal parts.
- Centroid: The point of concurrency of the medians of a triangle.
- Circumcenter: The point of concurrency of the perpendicular bisectors of the sides of a triangle.
- Coincidental: Two equivalent linear equations overlap when graphed.
- **Dilation**: Transformation that changes the size of a figure, but not the shape.
- Equiangular: The property of a polygon whose angles are all congruent.
- Equilateral: The property of a polygon whose sides are all congruent.
- Exterior Angle of a Polygon: an angle that forms a linear pair with one of the angles of the polygon.
- Incenter: The point of concurrency of the bisectors of the angles of a triangle.
- Intersecting Lines: Two lines in a plane that cross each other. Unless two lines are coincidental, parallel, or skew, they will intersect at one point.
- Linear Pair: Adjacent, supplementary angles. Excluding their common side, a linear pair forms a straight line.
- Measure of each Interior Angle of a Regular n-gon:  $\frac{180^{\circ}(n-2)}{n-2}$
- **Orthocenter:** The point of concurrency of the altitudes of a triangle.
- Plane: One of the basic undefined terms of geometry. Traditionally thought of as going on forever in all directions (in two-dimensions) and is flat (i.e., it has no thickness).
- Reflection: A transformation that "flips" a figure over a line of reflection

oncurrncypoints.html

- Isosceles Triangles: http://mathbitsnote book.com/Geometr y/SegmentsAnglesTr iangles/SATIsosceles .html
- □ Constructions:

http://www.mathsis fun.com/geometry/ constructions.html

- **Reflection Line:** A line that is the perpendicular bisector of the segment with endpoints at a pre-image point and the image of that point after a reflection.
- **Regular Polygon:** A polygon that is both equilateral and equiangular.
- **Remote Interior Angles of a Triangle:** the two angles non-adjacent to the exterior angle.
  - **Rotation:** A transformation that turns a figure about a fixed point through a given angle and a given direction.
  - Same-Side Interior Angles: Pairs of angles formed when a third line (a transversal) crosses two other lines. These angles are on the same side of the transversal and are between the other two lines. When the two other lines are parallel, same-side interior angles are supplementary.
  - Same-Side Exterior Angles: Pairs of angles formed when a third line (a transversal) crosses two other lines. These angles are on the same side of the transversal and are outside the other two lines. When the two other lines are parallel, same-side exterior angles are supplementary.
  - Scale Factor: The ratio of any two corresponding lengths of the sides of two similar figures.
  - Similar Figures: Figures that have the same shape but not necessarily the same size.
  - **Skew Lines:** Two lines that do not lie in the same plane (therefore, they cannot be parallel or intersect).
  - Sum of the Measures of the Interior Angles of a Convex Polygon:  $180^{\circ}(n-2)$ .
  - **Transformation**: The mapping, or movement, of all the points of a figure in a plane according to a common operation.
  - **Translation:** A transformation that "slides" each point of a figure the same distance in the same direction
  - Transversal: A line that crosses two or more lines.
  - Vertical Angles: Two nonadjacent angles formed by intersecting lines or segments. Also called opposite angles.

Try http://intermath.coe.uga.edu/dictnary/homepg.asp or http://www.amathsdictionaryforkids.com/ for further examples.



## Example 2

What theorem would prove these 2 triangles congruent?



### Example 3

Construct a regular hexagon inside of a circle.

Example 1: Yes these 2 triangles are similar because their sides are proportional. The scale factor of the dilation from the smaller triangle to the larger triangle is 2.

### <u>Example 2:</u> ASA because $\overline{KT} \cong \overline{TK}$ and $\angle IKT \cong \angle EKT$ ; $\angle ITK \cong \angle ETK$ Example 3:

We start with a line segment AF. This will become one side of the hexagon. Because we are constructing a regular hexagon, the other five sides will have this length also.	۸	<ol> <li>Move the compass on to A and draw an arc across the circle. This is the next vertex of the hexagon.</li> </ol>	
2. From points A and F, draw two arcs so that	,,	<ol><li>Move the compass to this arc and draw an arc across the circle to create the next vertex.</li></ol>	
they intersect. Mark this as point O. This is the center of the hexagon's circumcircle.	A		
<ol> <li>Move the compass to O and draw a circle.</li> <li>This is the hexagon's circumcircle - the circle that passes through all six vertices</li> </ol>		<ol> <li>Continue in this way until you have all six vertices. (Four new ones plus the points A and F you started with.)</li> </ol>	
		<ol> <li>Draw a line between each successive pairs of vertices.</li> </ol>	
		<ol> <li>Done. These lines form a regular hexagon where each side is equal in length to AF.</li> </ol>	A

### Key



# Unit 3: Right Triangle Trigonometry

Geometry

#### Dear Parents,

Below is information regarding Unit 3, Right Triangle Trigonometry.

### In this unit students will:

- explore the relationships that exist between sides and angles of right triangles
- build upon their previous knowledge of similar triangles and of the Pythagorean Theorem to determine the side length ratios in special right triangles
- understand the conceptual basis for the functional ratios sine and cosine
- explore how the values of these trigonometric functions relate in complementary angles
- to use trigonometric ratios to solve problems
- develop the skills and understanding needed for the study of many technical areas
- build a strong foundation for future study of trigonometric functions of real numbers



Right Triangle Trigonometry Vocabulary Terms/Properties
<u>Complementary Angles</u> : two angles whose sum is 90°
sine of $\theta - \sin(\theta) = \frac{1}{2}$ length of opposite side
length of the hypotenuse
cosine of $\beta = \cos(\beta) = \frac{1}{2}$ length of adjacent side
<u>cosine of o</u> length of the hypotenuse
tancent of $\theta = tan(\theta)$ - length of opposite side
length of adjacent side
Properties, theorems & corollaries:
1) 30°-60°-90° triangles pattern: hypotenuse, shorter leg, longer leg= 2a, a, a $\sqrt{3}$
2) 45°-45°-90° triangles pattern: leg lengths equal & hypotenuse is $\sqrt{2}$ times the length of a leg
3) Pair of complementary angles in a rt. triangle, the sine of one angle is the cosine of its complement.
4) Pair of complementary angles in a rt. triangle, the tangent of one angle is the reciprocal of the tangent of its complement.

For examples & help with vocabulary, visit: <a href="http://intermath.coe.uga.edu/">http://intermath.coe.uga.edu/</a>

## Web Resources

- <u>https://mathbitsnotebook.com/Geometry/RightTriangles/RT306090.html</u> special right triangles
- <u>https://www.cliffsnotes.com/study-guides/geometry/right-triangles/special-right-triangles</u> special right triangles
- http://www.beaconlearningcenter.com/documents/1688\_01.pdf -special right triangles
- <u>http://www.purplemath.com/modules/basirati.htm</u> -trigonometry ratios
- <u>http://www.themathlab.com/toolbox/geometry%20stuff/trigratios.htm</u> -trig. table
- <u>http://hotmath.com/hotmath\_help/topics/trigonometric-ratios.html</u> -trig ratio short notes

### **Practice**

1. What are the measurements of x, y, q and z?



2. A man is walking his dog on level ground in a straight line with the dog's favorite tree. The angle of elevation from the man's present position to the top of a nearby telephone pole is 30°. The angle of elevation from the tree to the top of the telephone pole is 45°. If the telephone pole is 40 feet tall, how far is the man with the dog from the tree? Express answer to the nearest tenth of a foot.



- 3. Find the **exact** value of:  $\cos 60^\circ + \sin 30^\circ \tan 45^\circ$ .
- 4. Find to the *nearest degree*, the measure of an acute angle formed by the x-axis and the line containing the points (4,3) and (8,9).
- 5. In  $\triangle ABC$ , m<B = 80°, m<C = 34° and a = 16. Find the length of b to the nearest tenth



### Answers:

- 1. x=12√3; y=12; q=6; z=18
- 2. **29.3 ft**
- з. О
- 4. 56°
- 5. **≈17.2**



# <u>Geometry Unit 4</u> Circles & Volume

## References

Textbook Connection: Analytic Geometry: Unit 3 Modules 11-12 (HMH)

HMH Digital Textbook http://my.hrw.com

### **Helpful Links:**

#### Lesson on angles: http://www.brightstor m.com/math/geometr y/circles/inscribedangles/

Lesson on segments: https://mathbitsnoteb ook.com/Geometry/Ci rcles/CRSegmentRules. html

Lesson on Constructions: http://www.math.nms

rseMaterials/Inscribed Triangles.html

Lesson on
 Constructions:

http://www.mathopen ref.com/consttangents .html

Lesson on Volume: http://www.mathexpr ession.com/volumeformulas.html

 http://cms.gavirtualsc hool.org/Shared/Math /GSEGeometries17/GS EGeometry\_Circlesand VolumePart1\_Shared/i ndex.html

http://cms.gavirtualsc hool.org/Shared/Math /GSEGeometries17/GS EGeometry\_Circlesand VolumePart2\_Shared/i ndex.html

## Dear Parents

In this unit, students will explore and understand parts of a circle and their relationship to each other. Students will formalize an understanding of the development of volume formulas and use them at an application level.

## Concepts Students will Use & Understand

- Understand and apply theorems about circles.
- Construct the inscribed & circumscribed circles of a triangle and prove properties of angles for a quadrilateral inscribed in a circle.
- Construct a tangent line from a point outside a given circle to the circle.
- Find arc lengths and areas of sectors of circles.
- Explain volume formulas and use them to solve problems

## Vocabulary

**Central Angle:** an angle whose vertex is at the center of a circle **Chord:** a segment whose endpoints are on a circle **Circumcenter:** The point of intersection of the perpendicular bics

**Circumcenter:** The point of intersection of the perpendicular bisectors of the sides of a given triangle; the center of the circle circumscribed about a given triangle

**Circumscribed Circle:** a circle containing an inscribed polygon; for this unit the polygon will be a triangle and so the center of the circle will be the circumcenter of the triangle **Inscribed:** an inscribed planar shape or solid is one that is enclosed by and "fits snugly" inside

**Inscribed:** an inscribed planar shape or solid is one that is enclosed by and "fits snugly" inside another geometric shape or solid

**Inscribed Angle:** an angle whose vertex is on the circle and whose sides contain chords of a circle **Inscribed Circle:** a circle enclosed in a polygon, where every side of the polygon is a tangent to the circle; specifically for this unit the polygon will be a triangle and so the center of the Inscribed Circle is the incenter of the triangle

Inscribed Polygon: a polygon whose vertices all lie on a circle

Point of Tangency: the point where a tangent line touches a circle

**Secant Line:** a line in the plane of a circle that intersects a circle at exactly two points **Secant Segment:** a segment that contains a chord of a circle and has exactly one endpoint outside of the circle

**Tangent Line:** a line in the plane of a circle that intersects a circle at only one point, the point of tangency

**Cavalieri's Principle:** a method, with formula given below, of finding the volume of any solid for which cross-sections by parallel planes have equal areas; this includes, but is not limited to, cylinders and prisms

Try http://intermath.coe.uga.edu/dictnary/homepg.asp or http://www.amathsdictionaryforkids.com/ for further examples.

### Sample Practice Problems



### Example 3

The two cones have the same radius. How much greater is the volume of the taller cone than the shorter cone?



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Example 1: x=4

Example 2: Find fractional portion of the circle by using 100/360. Area sector=100/360(pi)(6<sup>2</sup>). Area of the sector is 31.4 in.<sup>2</sup>

**Example 3:** The larger cone is approximately 469.14 cubic inches greater in volume.



# <u>Geometry: Unit 5</u> Geometric & Algebraic Connections

### References

#### Textbook:

- HMH Analytic Geometry, Unit 6
- HMH Coordinate
   Algebra, Unit 6
- HMH Advanced Algebra, Unit 6

### **Online Access:**

http://www.my.hrw.com

### **Helpful Links:**

- Circle Equations: http://www.purplem ath.com/modules/sq rcircle.htm
- Area and Perimeter on a Grid:

https://mathbits notebook.com/G eometry/Coordin ateGeometry/CG Area.html

- Distance Formula: https://mathbitsnote book.com/Geometry /CoordinateGeometr y/CGdistance.html
- Partition a Line Segment:

https://mathbitsnote book.com/Geometry /CoordinateGeometr y/CGdirectedsegmen ts.html

 Coordinate Geometry Proofs: https://mathbitsnote book.com/Geometry /CoordinateGeometr y/CGShowProofs.htm

#### Dear Parents,

Students will use the concepts of distance, midpoint, and slope to verify algebraically geometric relationships of figures in the coordinate plane (triangles, quadrilaterals, and circles). Students will solve problems involving parallel and perpendicular lines, perimeters and areas of polygons, and the partitioning of a segment in a given ratio. Students will derive the equation of a circle and model real-world objects using geometric shapes and concepts.

#### **Concepts Students will Use & Understand**

- prove the slope relationship that exists between parallel lines and between perpendicular lines and then use those relationships to write the equations of lines
- extend the Pythagorean Theorem to the coordinate plane
- develop and use the formulas for the distance between two points and for finding the point that partitions a line segment in a given ratio
- revisit definitions of polygons while using slope and distance on the coordinate plane
- use coordinate algebra to determine perimeter and area of defined figures
- use Algebra to model Geometric ideas
- spend time developing equations from geometric definition of circles
- address equations in standard and general forms
- graph by hand and by using graphing technology
- develop the idea of algebraic proof in conjunction with writing formal geometric proofs

#### Vocabulary

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- Distance Formula:  $d = \sqrt{(x_2 x_1)^2 + (y_2 y_1)^2}$
- Formula for finding the point that partitions a directed segment AB at the ratio of a : b from A(x<sub>1</sub>, y<sub>1</sub>) to B(x<sub>2</sub>, y<sub>2</sub>):

$$x_1 + \frac{a}{a+b}(x_2 - x_1), \ y_1 + \frac{a}{a+b}(y_2 - y_1)$$

r 
$$\left(\frac{a}{a+b}(x_2-x_1)+x_1, \frac{a}{a+b}(y_2-y_1)+y_1\right)$$

or 
$$\left(\frac{bx_1 + ax_2}{b + a}, \frac{by_1 + ay_2}{b + a}\right) \leftarrow$$
 weighted average approach

- **Center of a Circle**: The point inside the circle that is the same distance from all of the points on the circle.
- **Circle:** The set of all points in a plane that are the same distance, called the radius, from a given point, called the center. Standard form:  $(x h)^2 + (y k)^2 = r^2$

- **Diameter**: The distance across a circle through its center. The line segment that includes the center and whose endpoints lie on the circle.
- **Pythagorean Theorem**: A theorem that states that in a right triangle, the square of the length of the hypotenuse equals the sum of the squares of the lengths of the legs.
- **Radius:** The distance from the center of a circle to any point on the circle. Also, the line segment that has the center of the circle as one endpoint and a point on the circle as the other endpoint.
- Standard Form of a Circle:  $(x h)^2 + (y k)^2 = r^2$ , where (h,k) is the center and r is the radius.

### **Sample Practice Problems**

#### Example 1:

Write the standard form of the equation of a circle that passes through the given point (7,-4) and whose center is at the origin.

#### Example 2:

Show that A(2, -1), B(1, 3), C(6, 5), and D(7, 1) are the vertices of a parallelogram.



#### Key :

**Example 1**:  $x^2 + y^2 = 65$ **Example 2**:

#### SOLUTION

There are many ways to solve this problem.

**Method 1** Show that opposite sides have the same slope, so they are parallel.

Slope of  $\overline{AB} = \frac{3 - (-1)}{1 - 2} = -4$ Slope of  $\overline{CD} = \frac{1 - 5}{7 - 6} = -4$ Slope of  $\overline{BC} = \frac{5 - 3}{6 - 1} = \frac{2}{5}$ Slope of  $\overline{DA} = \frac{-1 - 1}{2 - 7} = \frac{2}{5}$ 

 $\overline{AB}$  and  $\overline{CD}$  have the same slope so they are parallel. Similarly,  $\overline{BC} \parallel \overline{DA}$ .

Method 2 Show that opposite sides have the same length.

$$AB = \sqrt{(1-2)^2 + [3-(-1)]^2} = \sqrt{17}$$
$$CD = \sqrt{(7-6)^2 + (1-5)^2} = \sqrt{17}$$
$$BC = \sqrt{(6-1)^2 + (5-3)^2} = \sqrt{29}$$
$$DA = \sqrt{(2-7)^2 + (-1-1)^2} = \sqrt{29}$$

AB  $\cong \overline{CD}$  and  $\overline{BC} \cong \overline{DA}$ . Because both pairs of opposite sides are congruent, *ABCD* is a parallelogram.

Method 3 Show that one pair of opposite sides is congruent and parallel.

Find the slopes and lengths of  $\overline{AB}$  and  $\overline{CD}$  as shown in Methods 1 and 2.

Slope of 
$$\overline{AB}$$
 = Slope of  $\overline{CD}$  = -4

$$AB = CD = \sqrt{17}$$

- $\overrightarrow{AB}$  and  $\overrightarrow{CD}$  are congruent and parallel, so *ABCD* is a parallelogram.
- Because opposite sides are parallel, ABCD is a parallelogram.



## **Geometry Unit 6** Applications of Probability

## References

#### **Textbook Connection:**

Holt McDougal Analytic Geometry Unit 7 **Online Access:** http://www.my.hrw.com

#### Helpful Links:

#### □ Set Notation:

- Venn Diagrams and Set Notation:
- Conditional Probability:
- Two way Frequency Tables:
- Conditional Probability and Independence:

### **Dear Parents**

In this unit, students will take their previously acquired knowledge of probability for simple and compound events and expand that to include conditional probabilities (events that depend upon and interact with other events) and independence. Students will be exposed to elementary set theory and notation (sets, subsets, intersection and unions). Finally, students will use their knowledge of conditional probability and independence to make determinations on whether or not certain variables are independent.

### **Concepts Students will Use & Understand**

- Use set notation as a way to algebraically represent complex networks of events or real world objects.
- Represent everyday occurrences mathematically through the use of unions, intersections, complements and their sets and subsets.
- Use Venn Diagrams to represent the interactions between different sets, events or probabilities.
- Find conditional probabilities by using a formula or a two-way frequency table.
- Understand independence as conditional probabilities where the conditions are irrelevant.
- Analyze games of chance, business decisions, public health issues and a variety of other parts of everyday life can be with probability.
- Model situations involving conditional probability with two-way frequency tables and/or Venn Diagrams.
- Confirm independence of variables by comparing the product of their probabilities with the probability of their intersection.

### Vocabulary

- **Complement:** Given a set A, the complement of A, denoted A or A', is the set of elements that are not members of A.
- **Conditional Probability:** The probability of an event A, given that another event, B, has already occurred; denoted P(A | B).
- **Dependent Events:** Two or more events in which the outcome of one event affects the outcome of the other event or events.
- Element: A member or item in a set.
- Independent Events: Events whose outcomes do not influence each other.
- Intersection of Sets: The set of all elements contained in all of the given sets, denoted ∩.
- Outcome: A possible result of an experiment.
- Sample Space: The set of all possible outcomes from an experiment.
- **Set:** A collection of numbers, geometric figures, letters, or other objects that have some characteristic in common.
- Subset: a set in which every element is also contained in a larger set.
- Union of Sets: The set of all elements that belong to at least one of the given two or more sets denoted ∪.
- Venn Diagram: A picture that illustrates the relationship between two or more sets.

### **Sample Practice Problems:**



2. Suppose a study of speeding violations and drivers who use car phones produced the following fictional data:

	Speeding violation in the last year	No speeding violation in the last year	Total
Car phone user	25	280	305
Not a car phone user	45	405	450
Total	70	685	755

- P(person is a car phone user) a.
- b. P(person had no violation in the last year)
- P( person had no violation in the last year AND was a car phone user) c.
- d. P(person is a car phone user GIVEN that they had a violation in the past year)
- If there is a 10% chance that the moon will be in the Seventh House and Jupiter will also align with Mars, and a 25% chance that Jupiter 3. will align with Mars, then what is the probability that the Moon is in the Seventh House given that Jupiter aligns with Mars?

#### Solutions:

**a**  $A = \{b, d, e, h\}$  **b**  $B = \{e, f, h, i, j\}$ **c**  $A' = \{a, c, f, g, i, j, k\}$  **d**  $B' = \{a, b, c, d, g, k\}$ 1. •  $A \cap B = \{e, h\}$  f  $A \cup B = \{b, d, e, f, h, i, j\}$ **g**  $(A \cup B)' = \{a, c, g, k\}$ **h**  $A' \cup B' = \{a, b, c, d, f, g, i, j, k\}$ 

 $\frac{\text{number of car phone users}}{\text{tar in study}} = \frac{305}{755}$ 2.

b. 
$$\frac{\text{number that had no violation}}{\text{total number in study}} = \frac{685}{755}$$

280755 c.

> 25 70 (The sample space is reduced to the number of persons who had a violation.)

3. Let: M=The Moon is in the Seventh House and J= Jupiter aligns with Mars, then

 $M|J) = P(M \cap J)/P(J) = .10/.25 = 0.4$ 



Pre-Calculus Teaching & Learning Framework									
	Sen	nester 1				Semester 2			
<b>Unit 1</b> 4 weeks	<b>Unit 2</b> 5 weeks	<b>Unit 3</b> 5 weeks	<b>Unit 4</b> 4 weeks	Unit 5 3 weeks	Unit 6 4 weeks	<b>Unit 7</b> 4 weeks	Unit 8 3 weeks	<b>Unit 9</b> 4 weeks	
Matrices	Conics	Introduction to Trigonometric Functions	Trigonometric Functions	Trigonometric Identities	Trigonometry of General Triangles	Vectors	Probability	*Sequences & Series Review	
MGSE9-12.N.VM.6 (Use matrices for data) MGSE9-12.N.VM.7 (Multiply matrices) MGSE9-12.N.VM.8 (Add, subtract & multiply matrices) MGSE9-12.N.VM.9 (Properties & multiplication of matrices) MGSE9-12.N.VM.10 (Zero & identity matrices) MGSE9-12.N.VM.12 (2x2 matrices & transformations) MGSE9-12.A.REI.8 (Systems & matrices) MGSE9-12.A.REI.9 (Inverse of a matrix)	MGSE9-12.G.GPE.2 (Derive the equation of a parabola) MGSE9-12.G.GPE.3 (Derive the equations of ellipses & hyperbolas) MGSE9-12.A.REI.7 (Solve a system of linear & quadratic equations)	MGSE9-12.F.IF.4 (Multiple representations with characteristics & key features) MGSE9-12.F.IF.7 (Algebraic to graphs ) MGSE9-12.F.IF.7e (Graph trig. functions) MGSE9-12.F.TF.1 (Radian measures) MGSE9-12.F.TF.2 (Unit circle) MGSE9-12.F.TF.5 (Periodic phenomena) MGSE9-12.F.TF.8 (Pythagorean identity)	MGSE9-12.F.BF.4 (Inverse functions) MGSE9-12.F.BF.4d (Invertible functions) MGSE9-12.F.TF.3 (Sine, cosine & tangent) MGSE9-12.F.TF.4 (Symmetry & periodicity) MGSE9-12.F.TF.6 (Restricted domain) MGSE9-12.F.TF.7 (Inverse functions & modeling)	MGSE9-12.F.TF.9 (Prove addition, subtraction, double and half- angle formulas) MGSE9-12.F.TF.8 (Pythagorean identity) MGSE9-12.F.TF.4 (Symmetry & periodicity) *CSE9-12.A.REI.1 (Solve Trigonometric Equations)	MGSE9- 12.G.SRT.9 (Derive the area of a triangle) MGSE9- 12.G.SRT.10 (Prove Laws of Sines & Cosines) MGSE9- 12.G.SRT.11 (Apply Laws of Sines & Cosines)	MGSE9-12.N.CN.3 (Conjugates of complex numbers) MGSE9-12.N.CN.4 (Complex #'s on complex planes) MGSE9-12.N.CN.5 (Addition, subtraction, multiplication & conjugation of complex #'s geometrically) MGSE9-12.N.CN.6 (Distance in the complex plane) MGSE9-12.N.CN.1 (Magnitude & direction) MGSE9-12.N.VM.2 (Components of a vector) MGSE9-12.N.VM.3 (Velocity) MGSE9-12.N.VM.3 (Velocity) MGSE9-12.N.VM.3 (Scalar multiplication using vectors & compute the magnitude) MGSE9-12.N.VM.11 (Multiple a vector by a matrix	MGSE9-12.S.CP.8 (General multiplication rule) MGSE9-12.S.CP.9 (Permutations & Combinations) MGSE9-12.S.MD.1 (Graph probability distributions) MGSE9-12.S.MD.2 (Calculate the expected value) MGSE9-12.S.MD.3 (Develop a probability distribution- theoretical) MGSE9-12.S.MD.4 (Develop a probability distribution- empirically- expected value) MGSE9- 12.S.MD.5,a,b (Expected values & expected payoff) MGSE9-12.S.MD.6 (Fair decisions) MGSE9-12.S.MD.7 (Probability concepts)	CSE.9-12.N.SEQ.1 a-i (recognize, formulate, & use sequence and series) <u>Review:</u> All standards by differentiating for student needs	
	These units w	ere written to build upon co All ur	ncepts from prior units, so l nits will include the Mathem	ater units contain task atical Practices and in	s that depend upon the dicate skills to maintain	concepts addressed in e	arlier units.		

NOTE: Mathematical standards are interwoven and should be addressed throughout the year in as many different units and topics as possible in order to stress the natural connections that exist among mathematical topics. \* denotes enrichment standards in preparation for AP Calculus: CSE-Cobb Standards of Excellence

Grades 9-12 Key: Algebra Strand: SSE = Seeing Structure in Expressions, APR = Arithmetic with Polynomial and Rational Expressions, CED = Creating Equations, REI = Reasoning with Equations and Inequalities Functions Strand: IF = Interpreting Functions, LE = Linear and Exponential Models, BF = Building Functions, TF = Trigonometric Functions

Geometry Strand: CO = Congruence, SRT = Similarity, Right Triangles, and Trigonometry, C = Circles, GPE = Expressing Geometric Properties with Equations, GMD = Geometric Measurement and Dimension,

### **Cobb County School District**

MG = Modeling with Geometry

Statistics and Probability Strand: ID = Interpreting Categorical and Quantitative Data, IC = Making Inferences and Justifying Conclusions, CP = Conditional Probability and the Rules of Probability, MD = Using Probability to Make Decisions



Pre-Calculus Teaching & Learning Framework									
	Block Schedule								
<b>Unit 1</b> 2 weeks	<b>Unit 2</b> 2.5 weeks	Unit 3 2.5 weeks	<b>Unit 4</b> 2 weeks	Unit 5 1.5 weeks	Unit 6 2 weeks	<b>Unit 7</b> 2 weeks	Unit 8 1.5 weeks	Unit 9 2 weeks	
Matrices	Conics	Introduction to Trigonometric Functions	Trigonometric Functions	Trigonometric Identities	Trigonometry of General Triangles	Vectors	Probability	*Sequences & Series Review	
MGSE9-12.N.VM.6 (Use matrices for data) MGSE9-12.N.VM.7 (Multiply matrices) MGSE9-12.N.VM.8 (Add, subtract & multiply matrices) MGSE9-12.N.VM.9 (Properties & multiplication of matrices) MGSE9-12.N.VM.10 (Zero & identity matrices) MGSE9-12.N.VM.12 (2x2 matrices & transformations) MGSE9-12.A.REI.8 (Systems & matrices) MGSE9-12.A.REI.9 (Inverse of a matrix)	MGSE9-12.G.GPE.2 (Derive the equation of a parabola) MGSE9-12.G.GPE.3 (Derive the equations of ellipses & hyperbolas) MGSE9-12.A.REI.7 (Solve a system of linear & quadratic equations)	MGSE9-12.F.IF.4 (Multiple representations with characteristics & key features) MGSE9-12.F.IF.7 (Algebraic to graphs ) MGSE9-12.F.IF.7e (Graph trig. functions) MGSE9-12.F.TF.1 (Radian measures) MGSE9-12.F.TF.2 (Unit circle) MGSE9-12.F.TF.5 (Periodic phenomena) MGSE9-12.F.TF.8 (Pythagorean identity)	MGSE9-12.F.BF.4 (Inverse functions) MGSE9-12.F.BF.4d (Invertible functions) MGSE9-12.F.TF.3 (Sine, cosine & tangent) MGSE9-12.F.TF.4 (Symmetry & periodicity) MGSE9-12.F.TF.6 (Restricted domain) MGSE9-12.F.TF.7 (Inverse functions & modeling)	MGSE9-12.F.TF.9 (Prove addition, subtraction, double and half- angle formulas) MGSE9-12.F.TF.8 (Pythagorean identity) MGSE9-12.F.TF.4 (Symmetry & periodicity) *CSE9-12.A.REI.1 (Solve Trigonometric Equations)	MGSE9- 12.G.SRT.9 (Derive the area of a triangle) MGSE9- 12.G.SRT.10 (Prove Laws of Sines & Cosines) MGSE9- 12.G.SRT.11 (Apply Laws of Sines & Cosines)	MGSE9-12.N.CN.3 (Conjugates of complex numbers) MGSE9-12.N.CN.4 (Complex #'s on complex planes) MGSE9-12.N.CN.5 (Addition, subtraction, multiplication & conjugation of complex #'s geometrically) MGSE9-12.N.CN.6 (Distance in the complex plane) MGSE9-12.N.VM.1 (Magnitude & direction) MGSE9-12.N.VM.2 (Components of a vector) MGSE9-12.N.VM.3 (Velocity) MGSE9- 12.N.VM.4,a,b,c (Addition & subtraction) MGSE9- 12.N.VM.5,a,b (Scalar multiplication using vectors & compute the magnitude) MGSE9-12.N.VM.11 (Multiple a vector by a matrix	MGSE9-12.S.CP.8 (General multiplication rule) MGSE9-12.S.CP.9 (Permutations & Combinations) MGSE9-12.S.MD.1 (Graph probability distributions) MGSE9-12.S.MD.2 (Calculate the expected value) MGSE9-12.S.MD.3 (Develop a probability distribution- theoretical) MGSE9-12.S.MD.4 (Develop a probability distribution- empirically- expected value) MGSE9- 12.S.MD.5,a,b (Expected values & expected payoff) MGSE9-12.S.MD.6 (Fair decisions) MGSE9-12.S.MD.7 (Probability concepts)	CSE.9-12.N.SEQ.1 a-i (recognize, formulate, & use sequence and series) <u>Review:</u> All standards by differentiating for student needs	
	These units w	ere written to build upon co All ur	ncepts from prior units, so l hits will include the Mathem	ater units contain task atical Practices and inc	s that depend upon the dicate skills to maintain	concepts addressed in e	earlier units.		

NOTE: Mathematical standards are interwoven and should be addressed throughout the year in as many different units and topics as possible in order to stress the natural connections that exist among mathematical topics. \* denotes enrichment standards in preparation for AP Calculus: CSE-Cobb Standards of Excellence

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## Pre-Calculus Unit 1: Matrices

## References

### Dear Parents,

In this unit students will:

- represent and manipulate data using matrices
- define the order of a matrix as the number of rows by the number of columns
- add and subtract matrices and know these operations are possible only when the dimensions are equal
- recognize that matrix addition and subtraction are commutative
- multiply matrices by a scalar and understand the distributive and associative properties apply to matrices
- multiply matrices and know when the operation is defined
- recognize that matrix multiplication is not commutative
- understand and apply the properties of a zero matrix
- understand and apply the properties of an identity matrix
- find the determinant of a square matrix and understand that it is a nonzero value if and only if the matrix has an inverse
- use 2 X 2 matrices as transformations of a plane and determine the area of the plane using the determinant
- write a system of linear equations as a matrix equation and use the inverse of the coefficient matrix to solve the system
- write and use vertex-edge graphs to solve problems

### **Concepts Students will Use & Understand**

- Commutative Property
- Associative Property
- Distributive Property
- Identity Properties of Addition and Multiplication
- Inverse Properties of Addition and Multiplication
- Solving Systems of Equations Graphically and Algebraically

### Vocabulary

- **Determinant:** the product of the elements on the main diagonal minus the product of the elements off the main diagonal
- Dimensions or Order of a Matrix: the number of rows by the number of columns
- Identity Matrix: the matrix that has 1's on the main diagonal and 0's elsewhere
- Inverse Matrices: matrices whose product ( in both orders) is the Identity matrix
- Matrix: a rectangular arrangement of numbers into rows and columns
- Scalar: in matrix algebra, a real number is called a scalar
- Square Matrix: a matrix with the same number of rows and columns
- Zero Matrix: a matrix whose entries are all zeros

For further help:

http://www.teachers.ash.org.au/jeather/maths/dictionary.html http://intermath.coe.uga.edu/dictnary/homepg.asp http://www.amathsdictionaryforkids.com/

Textbook Connection: Glencoe PreCalculus Text: Chapter 6

Every student will receive a text copy and access to the online textbook resource: http://www.connected.mcgrawhill.com

#### Helpful Links:

#### Matrices

- Add/Subtract Matrices
- Multiplying Matrices
- Systems & Matrices

Real-world Cryptography

### **Properties**

Let a, b, and c be real numbers	ADDITION PROPERTIES	MULTIPLICATION PROPERTIES
COMMUTATIVE	$\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$	ab = ba
ASSOCIATIVE	(a + b) + c = a + (b + c)	(ab)c = a(bc)
IDENTITY	There exists a unique real number zero, 0, such that a + 0 = 0 + a = a	There exists a unique real number one, 1, such that a * 1 = 1 * a = a
INVERSE	For each real number a, there is a unique real number - a such that a + (-a) = (-a) + a = 0	For each nonzero real number er a, there is a unique real number $\frac{1}{a}$ such that $a(\frac{1}{a}) = (\frac{1}{a})a = 1$
	- a such that a + (-a) = (-a) + a = 0	number $\frac{1}{a}$ such that $a(\frac{1}{a}) = (\frac{1}{a})a = 1$

### Sample Problems

1. Find the dimensions:

 $\begin{bmatrix} -3 & 5\\ 4 & 1/4\\ -\pi & 0 \end{bmatrix}$  3 rows, 2 columns; Dimensions: 3 x 2

2. Two stores carry small, medium, and large sweatshirts. The table shows the inventory at the stores. Arrange the data in a matrix. Give the dimensions of the matrix.

Sweatshirt Inventory							
	Small	Medium	Large				
Store A	6	21	13				
Store B	16	32	28				
$r_{c} = \frac{1}{21}$							

 $\begin{bmatrix} 0 & 21 & 13 \\ 16 & 32 & 28 \end{bmatrix}$  The dimensions are 2 x 3

3. Multiply the following matrix:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix}$$
$$\begin{bmatrix} 58 & 64 \\ 139 & 154 \end{bmatrix}$$

- 4. Find the inverse of the following matrix:
  - $\begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$
  - $\begin{bmatrix}
     1 & 1 & 9 \\
     0 & 1 & 9
     \end{bmatrix}$
  - .0 1 3



5. What system of equations is represented by the matrix equation?

$$\begin{bmatrix} -41 & 1 & 0 \\ 1 & 50 & 1 \\ 67 & 4 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ -29 \end{bmatrix}$$

-41x + y = 1x + 50y + z = 767x + 4y = -29



# Pre-Calculus Unit 2: Conics

## References

Textbook Connection: Glencoe PreCalculus Text: Chapter 7

Every student will receive a text copy and access to the online textbook resource: http://my.hrw.com/

## Helpful Links:

### GA Virtual:

http://cms.gavirtualschool. org/Shared/Math/GSEPreca lculus/Conics/index.html

### **Conics Introduction:**

http://math.about.com/libr ary/blconic.htm

### Conic Sections:

http://www.sparknotes.co m/math/precalc/conicsecti ons/

### Linear & Quadratic Systems:

http://tutorial.math.lamar. edu/Classes/Alg/NonLinear Systems.aspx

## Dear Parents,

In this unit, students will build on standards from previous courses, students will derive the equations of conic sections (parabolas, ellipses, and hyperbolas). Students will solve systems of a linear and quadratic equation in two variables.

## **Concepts Students will Use & Understand**

- Derive the equation of a parabola, ellipse, and hyperbola.
- Solve a linear and quadratic system in two variables.

## Vocabulary

- **Cone**: A three dimensional figure with a circular or elliptical base and one vertex.
- **Coplanar**: Set of points, lines, rays, line segments, etc., that lie in the same plane.
- **Ellipse:** A curved line forming a closed loop, where the sum of the distances from two points (foci) to every point on the line is constant.
- **Focus:** one of the fixed points from which the distances to any point of a given curve, such as an ellipse or parabola, are connected by a linear relation.
- **Hyperbola:** A plane curve having two branches, formed by the intersection of a plane with both halves of a right circular cone at an angle parallel to the axis of the cone. It is the locus of points for which the difference of the distances from two given points is a constant.
- Locus of Points: A group of points that share a property.
   Plane: One of the basic undefined terms of geometry. A plane goes on forever in all directions (in two-dimensions) and is "flat" (i.e., it has no thickness). For further help:

http://www.teachers.ash.org.au/jeather/maths/dictionary.html http://intermath.coe.uga.edu/dictnary/homepg.asp http://www.amathsdictionaryforkids.com/

## **Sample Practice Problems**

1. Write the standard equation of a parabola with a vertex at the origin and with the directrix y = 4.

- 2. A parabola defined by the equations  $4x + y^2 6y = 9$  is translated 2 units up and 4 units to the left. Write the standard equation of the resulting parabola.
- 3. Mars orbits the Sun in an elliptical path whose minimum distance from the Sun is 129.5 million miles and whose maximum distance from the Sun is 154.4 million miles. The Sun represents one focus of the ellipse. Write the standard equation for the elliptical orbit of Mars around the Sun, where the center of the ellipse is at the origin.
- 4. Write the standard equation for the hyperbola with vertices at (0, -4) and (0, 4) and co-vertices at (-3, 0) and (3, 0)
- 5. Find the equations of the asymptotes and the coordinates of the vertices for the graph of  $\frac{y^2}{16} \frac{x^2}{36} = 1$

#### **Answers to Sample Practice Problems**

1. 
$$y = \frac{1}{4(-4)}x^2$$
 OR  $y = -\frac{1}{16}x^2$ 

2. 
$$x - \frac{1}{2} = -\frac{1}{4}(y - 5)^2$$

3.  $\frac{x^2}{20,149.8} + \frac{y^2}{19,994.8} = 1$ 

$$4. \quad y = \pm \sqrt{16\left(1 + \frac{x^2}{9}\right)}$$

5. The asymptotes are  $y = \pm \frac{2}{3}x$ . The vertices are (0, -4) and (0, 4)



# PreCalculus Unit 3: Introduction to Trigonometric Functions

## References

### Dear Parents,

Textbook Connection: Glencoe PreCalculus Text: Chapter 4

www.connected.mcgra w-hill.com

### Helpful Links:

- □ Learn Zillion Video
- Khan Academy Video
- Khan Academy Video
- Khan Academy Video
- Khan Academy Video

In this unit, students will be introduced to a basic understanding of trigonometric functions,
which will be further developed upon in Unit 2. Students will apply their understanding of right triangle trig, learned in Geometry, and apply it to the unit circle. Students will also learn how to graph trig functions along with identifying and understanding characteristics specific to trig functions.

## Concepts Students will Use & Understand

- Explain what is meant by the radian measure of an angle.
- Define the trigonometric functions in terms of a point on the unit circle.
- Be able to determine the trigonometric values of one of the special real numbers by using the reference number of that real number.
- Be able to graph a trigonometric function and identify its characteristics.
- Know what is meant by the amplitude, the period, and the phase shift of a trigonometric function.
- Be able to write an equation of a trigonometric function given the characteristics of that function.
  - Be able to explain why  $(\sin t)^2 + (\cos t)^2 = 1$  is an identity, and use it to solve problems.

## Vocabulary

- Standard Position
- Initial Side
- Terminal Side
- Co-terminal Angle
- Reference Angle
- Unit Circle
- Radian
- Subtended Arc
- Sinusoidal Function
- Midline
- Amplitude
- Period
- Frequency

#### For further help:

http://www.teachers.ash.org.au/jeather/maths/dictionary.html http://intermath.coe.uga.edu/dictnary/homepg.asp http://www.amathsdictionaryforkids.com/

### Example 1

Convert 78° to radians. Convert  $\frac{2\pi}{3}$  radians to degrees.

### Example 2

Find the coordinates of the point where the terminal side of a  $330^{\circ}$  angle intersects the unit circle.

### Example 3

What is the  $\cos \frac{7\pi}{6}$  ?

### Example 4

Graph the function  $f(x) = 2 \sin x$  over the restricted domain  $[-2\pi, 2\pi]$ .

Then describe the period, midline, amplitude, and frequency.



### Example 5

Write an equation to describe the graphed function.





# PreCalculus Unit 4: Trigonometric Functions

## References

### Dear Parents,

Building on standards from Unit 3, students extend their study of the unit circle and trigonometric functions. Students will create inverses of trigonometric functions and use the inverse functions to solve trigonometric equations that arise in real-world problems.

## Concepts Students will Use & Understand

- Build upon understanding of the trigonometric functions
- Use special right triangles to determine the x- and y-coordinates of angles on the unit circle.
- Investigate how the symmetry of the unit circle helps to extend knowledge to angles outside of the first quadrant
- Use the symmetry of the unit circle to define sine and cosine as even and odd functions
- Investigate inverse trigonometric function
- Use trigonometric inverses to solve equations and real-world problems.

## Vocabulary

**Co-terminal Angle:** Two angles are co-terminal if they are drawn in the standard position and both have their terminal sides in the same location.

**Even Function:** A function f is even if the graph of f is symmetric with respect to the y-axis. Algebraically, f is even if and only if f(-x) = f(x) for all x in the domain of f.

**Odd Function:** A function f is odd if the graph of f is symmetric with respect to the origin. Algebraically, f is odd if and only if f(-x) = -f(x) for all x in the domain of f.

**Reference Angle:** A reference angle for angle  $\theta$  is the positive acute angle made by the terminal side of angle  $\theta$  and the x-axis.

**Special Right Triangles:** Refers to the 45-45-90 and 30-60-90 right triangles **Terminal side of angle:** The initial side of an angle lies on the x-axis. The other side, known as the terminal side, is the one that can be anywhere and defines the angle. **Unit Circle:** A unit circle is a circle that has a radius of one unit.

For further help: http://www.teachers.ash.org.au/jeather/maths/dictionary.html http://intermath.coe.uga.edu/dictnary/homepg.asp http://www.amathsdictionaryforkids.com/

## Sample Practice Problems

Textbook Connection: Glencoe PreCalculus Text: Chapter 4 www.connected.mcgrawhill.com

Helpful Links:

GA Virtual Learning

http://cms.gavirtualschool.or g/Shared/Math/GSEPrecalcul us/TrigonometricFunctions/i ndex.html

#### Unit Circle Self-Assessment

http://www.talljerome.com/ NOLA/100528\_unitcircle.htm I

#### Unit Circle Formula

http://www.mathwarehouse. com/unit-circle/graph-andformula-unit-circle.php

#### Unit Rate & Trigonometric Ratios

http://www.mathsisfun.com/ algebra/trig-interactive-unitcircle.html

Regent's Prep: Working w/ Inversion Trig Functions http://www.regentsprep.org/regen ts/math/algtrig/att8/inversetrig2.ht m



$$sinx = -\frac{1}{2}$$



## Pre-Calculus Unit 5: Trigonometric Identities

## References

## Dear Parents,

In this unit students will:

- build upon their work with trigonometric identities with addition and subtraction formulas
- will look at addition and subtraction formulas geometrically
- prove addition and subtraction formulas
- use addition and subtraction formulas to solve problems

### **Concepts Students will Use & Understand**

- Demonstrate a method to prove addition or subtraction identities for sine, cosine, and tangent.
- Apply addition or subtraction identities for sine, cosine, and tangent.
- Use addition or subtraction identities to find missing values for sine, cosine and tangent functions.

### Vocabulary

- Addition Identity for Cosine: cos(x + y) = cos x cos y sin x sin y
  - Addition Identity for Sine: sin(x + y) = sin x cos y + cos x sin y
- Addition Identity for Tangent:  $tan(x + y) = \frac{tan x + tan y}{1 tan x tan y}$
- Double Angle Identity for Sine: sin(2x) = 2 sin x cos x
- Double Angle Identity for Cosine:

$$\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

- Double Angle Identity for Tangent:  $\tan 2x = \frac{2 \tan x}{1 \tan^2 x}$
- Half Angle Identity for Sine:  $\sin(\frac{x}{2}) = \pm \sqrt{\frac{1-\cos x}{2}}$
- Half Angle Identity for Cosine:  $\cos(\frac{x}{2}) = \pm \sqrt{\frac{1+\cos x}{2}}$
- Half Angle Identity for Tangent:  $\tan\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1-\cos x}{1+\cos x}} = \frac{\sin x}{1+\cos x} = \frac{1-\cos x}{\sin x}$
- **Identity**: an identity is a relation that is always true, no matter the value of the variable.
- Subtraction Identity for Cosine: cos(x y) = cos x cos y + sin x sin y
- Subtraction Identity for Sine: sin(x-y) = sin x cos y cos x sin y
- Subtraction Identity for Tangent:  $tan(x y) = \frac{tan x tan y}{1 + tan x tan y}$

#### For further help:

http://www.teachers.ash.org.au/jeather/maths/dictionary.html http://intermath.coe.uga.edu/dictnary/homepg.asp http://www.amathsdictionaryforkids.com/

Textbook Connection: Glencoe PreCalculus Text: Chapter 5

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# Helpful Links:

## Addition Identities

- Khan Academy
- Addition & Double Angle for Sine/Cosine
- Double & Half Angles
- All Identities
- □ All Identities Practice

## **Sample Practice Problems**

- 1. Find the exact value of each trigonometric expression.
  - a. cos 75°
  - b. sin (-210°)
  - c.  $\tan \frac{\pi}{12}$
- 2. Find the exact value of each expression.
  - a.  $sin15^{\circ}cos75^{\circ} + cos15^{\circ}sin75^{\circ}$
  - $\frac{tan48^{\circ}+tan12^{\circ}}{1-tan48^{\circ}tan12^{\circ}}$ b.
- 3. Simplify each expression.
  - a. *sin3ycosy* + *cos3ysiny*
  - b.  $\frac{tan5\theta+tan\theta}{dt}$
  - $tan 5 \theta tan \theta 1$
- 4. Find the values of sin  $2\theta$ , cos  $2\theta$ , and tan  $2\theta$  for the given value and interval.

$$tan\theta = \sqrt{3}, (0, \frac{\pi}{2})$$

- 5. Solve each equation on the interval  $[0, 2\pi]$ .
  - a.  $\sin 2\theta = \cos \theta$
  - b.  $\tan 2\theta \tan 2\theta \tan^2 \theta = 2$
- 6. Find the exact value of each expression.
  - a. sin 67.5°
  - b.  $cos \frac{\pi}{12}$
- 7. Solve each equations on the interval  $[0, 2\pi]$ .

a. 
$$sin\frac{\theta}{2} + cos\theta = 1$$
  
b.  $tan\frac{\theta}{2} = sin\frac{\theta}{2}$ 

Answer Key

$1a.\frac{\sqrt{6}-\sqrt{2}}{4}$	1b. ½	1c. 2 $-\sqrt{3}$
2a. 1	2b. √3	
3a. sin4y	3b. <i>—tan6θ</i>	
$4.\frac{\sqrt{3}}{2}; -\frac{1}{2}; -\sqrt{3}$		
5a. $\frac{\pi}{6}$ , $\frac{\pi}{2}$ , $\frac{5\pi}{6}$ , $\frac{3\pi}{2}$	5b. $\frac{\pi}{4}$ , $\frac{5\pi}{4}$	
$6a.\frac{\sqrt{2+\sqrt{2}}}{2}$	6b. $\frac{\sqrt{2+\sqrt{3}}}{2}$	
7a. $0, \frac{\pi}{3}, \frac{5\pi}{3}$	7b. 0	



# <u>PreCalculus</u> Unit 6: Trigonometry of General Triangles

## References

## Dear Parents,

Textbook Connection: Glencoe PreCalculus Text: Chapter 4 www.connected.mcgrawhill.com

### Helpful Links:

#### GA Virtual Learning

http://cms.gavirtualschool.org/S hared/Math/GSEPrecalculus/Tri gOfGeneralTriangles/index.html

- http://ccssmath.org/? page\_id=2289
- http://ccssmath.org/? page\_id=2291
- http://ccssmath.org/? page\_id=2293
- http://www.shmoop.c om/common-corestandards/ccss-hs-gsrt-9.html
- http://www.shmoop.c om/common-corestandards/ccss-hs-gsrt-10.html
- http://www.shmoop.c om/common-corestandards/ccss-hs-gsrt-11.html
- https://www.engagen y.org/ccls-math/gsrt9
- https://www.engagen y.org/ccls-math/gsrt11

 https://www.opened.c om/search?category=s imilarity-righttriangles-andtrigonometry&grade= 10&grade\_group=high -schoolgeometry&standard=G .SRT.11&standard\_gro up=common-coremath Building on standards from Unit 1 and Unit 2, students will apply trigonometry to general triangles. Students will derive the trigonometric formula for the area of a triangle and prove and use the Laws of Sines and Cosines to solve problems.

## **Concepts Students will Use & Understand**

- Expand the use of trigonometric functions beyond right triangles into more general triangles.
- Develop the trigonometric formula for area of triangle.
- Use the Laws of Sines and Cosines to solve problems.

# Vocabulary

**Altitude of a Triangle:** The perpendicular distance between a vertex of a triangle and the side opposite that vertex. Sometimes called the height of a triangle. Also, sometimes the line segment itself is referred to as the altitude.

**Hinge Theorem:** If two sides of one triangle are congruent to two sides of another triangle, and the included angle of the first is larger than the included angle of the second, then the third side of the first triangle is longer than the third side of the second triangle. (Wikipedia)

Included Angle: The angle between two given sides of a triangle

**Law of Cosines:** The square of the length of any side of a triangle equals the sum of the squares of the lengths of the other two sides minus twice the product of the lengths of the other two sides and the cosine of the angle between them. (Swokowski, Cole)

**Law of Sines:** In any triangle, the ratio of the sine of an angle to the side opposite that angle is equal to the ratio of the sine of another angle to the side opposite that angle (Swokowski, Cole) **Oblique Triangle:** A triangle that is not a right triangle

Vertex of a Triangle: The common endpoint of the two legs that serve as the sides of a triangle

1	Law of Sines
Case: ASA or AAS	Case: SSA
$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$	$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
Law of C	osines
Case: SAS	Case: SSS
$a^2 = b^2 + c^2 - 2bc\cos A$	$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$
$b^2 = a^2 + c^2 - 2ac\cos B$	$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$
$c^2 = a^2 + b^2 - 2ab\cos C$	$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

## Sample Practice Problems

In triangle ABC find c if  $A = 36^{\circ}$ ,  $B = 101^{\circ}$ , and b = 42.7.



- b. About 40.2 units
- c. About 29.666 units
- Od. About 1.6 units

Two observers 3 miles apart and facing each other find that the angles of elevation of a balloon in the same vertical plane with themselves are 28° and 31° respectively. Find the distance from the balloon to the observer located at the 28° angle.

About	2.4	mi	les
	About	About 2.4	About 2.4 mi

- b. About 3.5 miles
- C. About 1.8 miles
- d. About 1.6 miles

Determine the number of possible solutions for traingle ABC, given  $A = 40^{\circ}$ , a = 7, and b = 9.

a. Two
 b. Three
 c. None

🔍 **d.** One

Given the following triange, find its area and express it rounded to three decimal places.



 $A = \frac{1}{2}abSinC$ 

From the figure, we know the following:

Substituting the values into the equation yields:

$$A = \frac{1}{2} 18 \cdot 12 \cdot Sin94^{\circ}$$

$$A = 107.736914 \text{ (Make sure your calculator is set to degree mode)}$$

$$A = 107.737$$



# Pre-Calculus Unit 7: Vectors

### References

### Textbook Connection: Glencoe PreCalculus Text: Chapter 8

Every student will receive a text copy and access to the online textbook resource: http://my.hrw.com/

# Helpful Links:

GA Virtual:

http://cms.gavirtualschool.org /Shared/Math/GSEPrecalculus /Vectors/index.html

### Teach Mathematics:

http://www.teachmathematic s.net/activities/dancingvectors.htm

### Better Explained:

http://betterexplained.com/ar ticles/vector-calculusunderstanding-the-dotproduct/

http://betterexplained.com/ar ticles/measure-any-distancewith-the-pythagoreantheorem/

### Faraday:

http://faraday.physics.utoront o.ca/PVB/Harrison/Flash/Vect ors/Add2Vectors.html

http://faraday.physics.utoront o.ca/PVB/Harrison/Flash/Vect ors/Add3Vectors.html http://faraday.physics.utoronto. ca/PVB/Harrison/Flash/Vectors/ Subtract2Vectors.html

### Dear Parents,

Students will extend their understanding of complex numbers and their operations through graphical representations. Students will perform operations on vectors and use the operations to represent various quantities.

### **Concepts Students will Use & Understand**

- Perform arithmetic operations with complex numbers.
- Represent complex numbers & their operations on the complex plane.
- Represent & model with vector quantities.
- Perform operations on vectors.
- Perform operations on matrices & use matrices in applications.

### Vocabulary

- **Vector:** A mathematical object that has both magnitude and direction. Vectors can be expressed as **v**, or  $\langle a, b \rangle$ , or as a directed line segment (arrow) in the plane.
- **Scalar:** A real number. A scalar has magnitude but not direction.
- Initial Point: The point at the "tail" of the arrow representing a vector.
   Often, the initial point is assumed to be (0, 0). This is the case in the notation (a, b).
- Terminal Point: The point at the "tip" of the arrow representing a vector.
- Magnitude of a Vector: The distance between a vector's initial and terminal points,

denoted  $||\mathbf{v}||$  or  $|\mathbf{v}|$ .  $||\mathbf{v}|| = ||\langle a, b \rangle|| = \sqrt{a^2 + b^2}$ Also called the length, norm, or absolute value of the vector.

- **Components of a Vector:** *a* and *b* in the vector  $\langle a, b \rangle$ .
- Parallel Vectors: Two or more vectors whose directions are the same or opposite.
- Equivalent Vectors: Two or more vectors that have the same direction and magnitude—i.e., whose representations are the same in the form (*a*, *b*). Note that equivalent vectors may not have the same initial and terminal points.
- **Zero Vector:** The vector  $\langle 0, 0 \rangle$ .
- **Resultant Vector:** The vector that results from adding two or more vectors.

**Tail-to-Head Representation:** A geometric representation of vector addition  $\mathbf{u} + \mathbf{v}$  wherein the initial point of  $\mathbf{v}$  is placed at the terminal point of  $\mathbf{u}$ . The vector beginning at the initial point of  $\mathbf{u}$  and ending at the (translated) terminal point of  $\mathbf{v}$  represents  $\mathbf{u} + \mathbf{v}$ .

 Parallelogram Representation / Parallelogram Rule: A geometric representation of vector addition u + v wherein a parallelogram is formed by placing the initial points of u and v at the same place and letting each vector represent the sides of





a parallelogram. The diagonal of the resulting parallelogram, starting at this shared initial point, represents  $\mathbf{u} + \mathbf{v}$ .

- Velocity: A vector whose magnitude is an object's speed (a scalar) and whose direction is the direction of the object's motion. Note that speed is a scalar—magnitude, no direction—whereas velocity tells us how fast an object is moving <u>and in what direction</u>.
- **Complex Plane:** A 2-dimensional representation of complex numbers established by a horizontal real axis and a vertical imaginary axis.
- Rectangular Form of a Complex Number: *a* + *bi*
- **cis**  $\vartheta$ : Shorthand for cos  $\vartheta + i \sin \vartheta$
- Polar Form of a Complex Number:  $r(\cos \vartheta + i \sin \vartheta) = r \operatorname{cis} \vartheta$
- Complex Conjugate of z = a + bi: z = a bi
- Modulus of a Complex Number: The distance between a number and 0 when plotted

on the complex plane:  $|z| = |a + bi| = \sqrt{zz} = \sqrt{a^2 + b^2}$ Also called absolute value or magnitude.

- Argument of *z*, arg(*z*): The angle—typically chosen in  $(-\pi, \pi]$ —formed by the positive-real axis and a segment connecting *z* to 0 in the complex plane.
- Re(z): a, the real part of the complex number z = a + bi
   Im(z): b, the coefficient of the imaginary part of the complex number z = a + bi

## **Sample Practice Problems**

- 1. Given:  $\left| \overrightarrow{u} \right| = 14, \left| \overrightarrow{\nu} \right| = 9, \& \overrightarrow{u} \parallel \overrightarrow{\nu}, find \left| \overrightarrow{u} + \overrightarrow{\nu} \right|.$ 23
- A ship leaving port sails for 100 miles in a direction 40° north of due east. Find the magnitude of the vertical and horizontal components.
   About 64 miles, about 77 miles
- 3. Find the component form of the given vector.



- What is the cross product of a = (1, 2, 3) and b = (4, 5, 6)?
   a × b = (-3, 6, -3)
- 5. Find the magnitude & direction of the sum of < 4, 5 > and < -1, 4 >. Magnitude:  $3\sqrt{10}$ ; Direction: 71.57°



# Pre-Calculus Unit 8: Probability

### References

### Textbook Connection: Glencoe PreCalculus Text: Chapter 0 & 11

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### **Helpful Links:**

#### GA Virtual:

http://cms.gavirtualschool.org /Shared/Math/GSEPrecalculus /Probability/index.html

#### Multiplication Rule:

https://www.youtube.com/watc h?v=Q\_7PR9kRXWs

#### Permutation & Combinations:

http://www.mathsisfun.com/co mbinatorics/combinationspermutations.html

http://regentsprep.org/Regents/ math/algtrig/ATS5/PCPrac.htm

#### Outcomes

https://www.khanacademy.org/ math/probability/randomvariables-topic/expectedvalue/e/expected\_value

http://www.mathsisfun.com/dat a/random-variables.html

https://www.khanacademy.org/ math/probability/randomvariables-

### Dear Parents,

Students will extend their learning about conditional probabilities, set theory and independent variables, by exploring permutations, combinations, expected value and random variables.

### **Concepts Students will Use & Understand**

- Calculate probabilities using the General Multiplication Rule and interpret the results in context
- Use permutations and combinations in conjunction with other probability methods to calculate probabilities of compound events and solve problems
- Define random variables, assign probabilities to its sample space, and graphically display the distribution of the random variable
- Calculate and interpret the expected value of random variables
- Develop the theoretical and empirical probability distribution and find expected values
- Set up a probability distribution for a random variable representing payoff values
- Make and explain in context decisions based on expected values.

### Vocabulary

Conditional Probability. 
$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Combinations. A combination is an arrangement of objects in which order does NOT

matter.  $_{n}C_{r} = \frac{n!}{r!(n-r)!}$ 

**Odds.** Typically expressed as a ratio of the likelihood that an event will happen to the likelihood that an event will not happen.

Permutations. An ordered arrangement of n objects. The order of the objects matters –

a different order creates a different outcome.  $_{n}P_{r} = \frac{n!}{(n-r)!}$ 

Sample Space. The set of all possible outcomes.

Expected Value: The mean of a random variable X is called the expected

value of X. It can be found with the formula  $\sum_{i=1}^{n} X_i P_i$  where  $P_i$  is the probability of the

value of  $X_i$ .

### **Sample Practice Problems**

1) How many ways could you select a chairman and a secretary for a committee of 10

people? 
$${}^{10}P_2 = \frac{10!}{(10-2)!} = \frac{10.9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{10.9}{1} = 90$$

2) How many ways could you select a committee of 3 people out of a group of 10 people?

$${}_{10}C_3 = \frac{10!}{3!(10-3)!} = \frac{10.9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{10.9 \cdot 8}{3 \cdot 2 \cdot 1} = 120$$

3) Paul is deciding whether or not to pay the parking meter when he is going to the movies. He knows that a parking ticket costs \$30 and he estimates that there is a 40% chance that the traffic police spot his car and write him a ticket. If he chooses to pay the meter it will cost 4 dollars and he will have a 0% chance of getting a ticket. Is it cheaper to pay the meter or risk the fine?

**Solution**: Since there are two possible scenarios, calculate the expected cost in each case. *Paying the meter*:  $4 \cdot 100\% = 4$ *Risking the fine*:  $0 \cdot 60\% + 30 \cdot 40\% = 12$ 

Risking the fine has an expected cost three times that of paying the meter.

4) What is the probability that the sum of two die will be greater than 8, given that the first die is 6?

Answer: 2/3

5) Suppose there are 12 multiple choice questions in a Mathematics class quiz. Each question has 5 possible answers and only 1 of them is correct. Find the probability of having 4 or less correct answers if a student attempts to answer every question at random.

Answer: 0.927



Use         Semester J         Unit 2         Unit 3         Unit 4         Unit 5         Unit 5         Unit 5         Unit 6         Unit 7         Unit 8         Unit 9         *Unit 10         3 weeks         4 weeks         9 weeks         2 weeks         Unit 7         Unit 8         Unit 9         *Unit 10         3 weeks         9 weeks         12 weeks         12 weeks<	Accelerated Pre-Calculus Teaching & Learning Framework										
Unit 1 3 weeksUnit 2 4 weeksUnit 3 4 weeksUnit 4 4 weeksUnit 5 3 weeksUnit 6 2 weeksUnit 7 4 weeksUnit 8 2 weeksUnit 9 4 weeks*Unit 10 3 weeksMatricesConicsIntroduction to Trigonometric FunctionsTrigonometric FunctionsTrigonometric FunctionsTrigonometric HotoricsTrigonometric FunctionsVectorsProbability 4 weeksInforences and ConclusionsPolar & S ParametricParametric ParametricParametric StructureParametric StructureParametric StructureParametric ParametricParamet			Semester	1				Sen	nester 2		
MatricesConicsIntroduction to Trigonometric FunctionsTrigonometric FunctionsTrigonometric IdentitiesTrigonometric rof General TrianglesProbabilityInferences and ConclusionsPolar & SSMGSE9- 12.N.VM.6, Use (User (triangles)MGSE9-12.F.F.F.4 (Multiple (torverse functions)MGSE9-12.F.F.F.4 (Inverse functions)MGSE9-12.F.F.7.4 (Inverse functions)MGSE9-12.F.F.7.4 (Inv	<b>Unit 1</b> 3 weeks	Unit 2 4 weeks	Unit 3 4 weeks	<b>Unit 4</b> 4 weeks	Unit 5 3 weeks	Unit 6 2 weeks	<b>Unit 7</b> 4 weeks	Unit 8 2 weeks	<b>Unit 9</b> 4 weeks	*Unit 10 3 weeks	*Unit 11 3 weeks
MGSE9- 12.N.VM.6MGSE9-12.F.R-4 (Multiple (Derive the aparabola) MGSE9-12.F.R-4MGSE9-12.F.R-4 (Inverse functions) MGSE9-12.F.R-4MGSE9-12.F.R-4 (Inverse functions) MGSE9-12.F.R-4MGSE9-12.F.R-4 (Inverse functions) MGSE9-12.F.R-4MGSE9-12.F.R-4 (Inverse functions) duble and half area of a (Complex function) duble and half area of a (Complex function)MGSE9-12.F.R-4 (Complex function) duble and half area of a (Complex function)MGSE9-12.F.R-4 (Complex function) (Complex function) duble and half (Complex function)MGSE9-12.F.R-4 (Inverse function) (Complex function)MGSE9-12.F.R-4 (Inverse function)MGSE9-12.F.R-4 (Inverse function)MGSE9-12.F.R-4 (Inverse function)MGSE9-12.F.R-4 (Inverse function)MGSE9-12.F.R-4 (Inverse function)MGSE9-12.F.R-4 	Matrices	Conics	Introduction to Trigonometric Functions	Trigonometric Functions	Trigonometric Identities	Trigonometr y of General Triangles	Vectors	Probability	Inferences and Conclusions from Data	Polar & Parametric Structure	Sequences & Series Review
transformati ons)expected payoff)MGSE9- 12.A.REI.8MGSE9- 12.S.MD.6(Systems & matrices)(Fair decisions)MGSE9- 12.S.REI.912.S.MD.7(Inverse of a matrix)(Probability concepts)	MGSE9- 12.N.VM.6 (Use matrices for data) MGSE9- 12.N.VM.7 (Multiply matrices) MGSE9- 12.N.VM.8 (Add, subtract & multiply matrices) MGSE9- 12.N.VM.9 (Properties & multiplicatio n of matrices) MGSE9- 12.N.VM.10 (Zero & identity matrices) MGSE9- 12.N.VM.12 (Zx2 matrices & transformati ons) MGSE9- 12.A.REI.8 (Systems & matrices) MGSE9- 12.A.REI.9 (Inverse of a matrix)	MGSE9- 12.G.GPE.2 (Derive the equation of a parabola) MGSE9- 12.G.GPE.3 (Derive the equations of ellipses & hyperbolas) MGSE9- 12.A.REI.7 (Solve a system of linear & quadratic equations)	MGSE9-12.F.IF.4 (Multiple representations with characteristics & key features) MGSE9-12.F.IF.7 (Algebraic to graphs ) MGSE9-12.F.IF.7e (Graph trig. functions) MGSE9-12.F.TF.1 (Radian measures) MGSE9-12.F.TF.2 (Unit circle) MGSE9-12.F.TF.5 (Periodic phenomena) MGSE9-12.F.TF.8 (Pythagorean identity)	MGSE9-12.F.BF.4 (Inverse functions) MGSE9-12.F.BF.4d (Invertible functions) MGSE9-12.F.TF.3 (Sine, cosine & tangent) MGSE9-12.F.TF.4 (Symmetry & periodicity) MGSE9-12.F.TF.6 (Restricted domain) MGSE9-12.F.TF.7 (Inverse functions & modeling)	MGSE9-12.F.TF.9 (Prove addition, subtraction, double and half- angle formulas) MGSE9-12.F.TF.8 (Pythagorean identity) MGSE9-12.F.TF.4 (Symmetry & periodicity) *CSE9-12.A.REI.1 (Solve Trigonometric Equations)	MGSE9- 12.G.SRT.9 (Derive the area of a triangle) MGSE9- 12.G.SRT.1 0 (Prove Laws of Sines & Cosines) MGSE9- 12.G.SRT.1 1 (Apply Laws of Sines & Cosines)	MGSE9-12.N.CN.3 (Conjugates of complex numbers) MGSE9-12.N.CN.4 (Complex #'s on complex planes) MGSE9-12.N.CN.5 (Addition, subtraction, multiplication & conjugation of complex #'s geometrically) MGSE9-12.N.CN.6 (Distance in the complex plane) MGSE9-12.N.CN.1 (Magnitude & direction) MGSE9-12.N.VM.2 (Components of a vector) MGSE9-12.N.VM.3 (Velocity) MGSE9- 12.N.VM.4,a,b,c (Addition & subtraction) MGSE9- 12.N.VM.5,a,b (Scalar multiplication using vectors & compute the magnitude) MGSE9-12.N.VM.11 (Multiple a vector by a matrix	MGSE9- 12.S.CP.8 (General multiplication rule) MGSE9- 12.S.CP.9 Permutations & Combinations) MGSE9- 12.S.MD.1 (Graph probability distributions) MGSE9- 12.S.MD.2 (Calculate the expected value) MGSE9- 12.S.MD.3 (Develop a probability distribution- theoretical) MGSE9- 12.S.MD.4 (Develop a probability distribution- theoretical) MGSE9- 12.S.MD.4 (Develop a probability distribution- empirically- expected value) MGSE9- 12.S.MD.5,a,b (Expected values & expected payoff) MGSE9- 12.S.MD.6 (Fair decisions) MGSE9- 12.S.MD.7 (Probability concepts)	MGSE9-12.S.ID.2 (Shape & data distribution) MGSE9-12.S.ID.4 (Fit to a normal distribution) MGSE9-12.S.IC.1 (Inferences from a random sample) MGSE9-12.S.IC.2 (Using simulations) MGSE9-12.S.IC.3 (Randomization) MGSE9-12.S.IC.4 (Population mean) MGSE9-12.S.IC.5 (Compare 2 treatments) MGSE9-12.S.IC.6 (Evaluate reports based on data)	CSE9- 12.N.CN.1 a- b (complex numbers in polar form) CSE.9- 12.F.POL1 a- e (understand & explore the polar coordinate plane) CSE.9- 12.F.POL2 a- e (explore & use parametric equations)	CSE.9- 12.N.SEQ.1 a-i (recognize, formulate, & use sequence and series) Review: All standards by differentiati ng for student needs

All units will include the Mathematical Practices and indicate skills to maintain

#### **Cobb County School District 2020-21**

**NOTE:** Mathematical standards are interwoven and should be addressed throughout the year in as many different units and topics as possible in order to stress the natural connections that exist among mathematical topics. \*denotes enrichment standards in preparation for AP Calculus

Grades 9-12 Key: Algebra Strand: SSE = Seeing Structure in Expressions, APR = Arithmetic with Polynomial and Rational Expressions, CED = Creating Equations, REI = Reasoning with Equations and Inequalities Functions Strand: IF = Interpreting Functions, LE = Linear and Exponential Models, BF = Building Functions, TF = Trigonometric Functions

Geometry Strand: CO = Congruence, SRT = Similarity, Right Triangles, and Trigonometry, C = Circles, GPE = Expressing Geometric Properties with Equations, GMD = Geometric Measurement and Dimension, MG = Modeling with Geometry

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	Accelerated Pre-Calculus Teaching & Learning Framework									
				Bloc	k Schedule					
Unit 1 1.5 weeks	<b>Unit 2</b> 2 weeks	Unit 3 2 weeks	<b>Unit 4</b> 2 weeks	Unit 5 1.5 weeks	<b>Unit 6</b> 1 weeks	<b>Unit 7</b> 2 weeks	Unit 8 1 weeks	Unit 9 2 weeks	*Unit 10 1.5 weeks	*Unit 11 1.5 weeks
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		These units were writte	n to build upon concepts	from prior units, so la	ter units contain	tasks that depend upon	the concepts add	lressed in earlier units.		

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### Cobb County School District 2020-21



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CSE=Cobb Standards of Excellence



# Acc. Pre-Calculus Unit 1: Matrices

## References

### Dear Parents,

In this unit students will:

- represent and manipulate data using matrices
- define the order of a matrix as the number of rows by the number of columns
- add and subtract matrices and know these operations are possible only when the dimensions are equal
- recognize that matrix addition and subtraction are commutative
- multiply matrices by a scalar and understand the distributive and associative properties apply to matrices
- multiply matrices and know when the operation is defined
- recognize that matrix multiplication is not commutative
- understand and apply the properties of a zero matrix
- understand and apply the properties of an identity matrix
- find the determinant of a square matrix and understand that it is a nonzero value if and only if the matrix has an inverse
- use 2 X 2 matrices as transformations of a plane and determine the area of the plane using the determinant
- write a system of linear equations as a matrix equation and use the inverse of the coefficient matrix to solve the system
- write and use vertex-edge graphs to solve problems

### **Concepts Students will Use & Understand**

- Commutative Property
- Associative Property
- Distributive Property
- Identity Properties of Addition and Multiplication
- Inverse Properties of Addition and Multiplication
- Solving Systems of Equations Graphically and Algebraically

### Vocabulary

- **Determinant:** the product of the elements on the main diagonal minus the product of the elements off the main diagonal
- Dimensions or Order of a Matrix: the number of rows by the number of columns
- Identity Matrix: the matrix that has 1's on the main diagonal and 0's elsewhere
- Inverse Matrices: matrices whose product ( in both orders) is the Identity matrix
- Matrix: a rectangular arrangement of numbers into rows and columns
- Scalar: in matrix algebra, a real number is called a scalar
- Square Matrix: a matrix with the same number of rows and columns
- Zero Matrix: a matrix whose entries are all zeros

For further help:

http://www.teachers.ash.org.au/jeather/maths/dictionary.html http://intermath.coe.uga.edu/dictnary/homepg.asp http://www.amathsdictionaryforkids.com/

Textbook Connection: Foerster PreCalculus Text: Chapter 13

Every student will receive a text copy and access to the online textbook resource: http://www.connected.mcgrawhill.com

### Helpful Links:

### Matrices

- □ Add/Subtract Matrices
- Multiplying Matrices
- Systems & Matrices

Real-world Cryptography

### **Properties**

Let a, b, and c be real numbers	ADDITION PROPERTIES	MULTIPLICATION PROPERTIES
COMMUTATIVE	$\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$	ab = ba
ASSOCIATIVE	(a + b) + c = a + (b + c)	(ab)c = a(bc)
IDENTITY	There exists a unique real number zero, 0, such that a + 0 = 0 + a = a	There exists a unique real number one, 1, such that a * 1 = 1 * a = a
INVERSE	For each real number a, there is a unique real number - a such that a + (-a) = (-a) + a = 0	For each nonzero real number er a, there is a unique real number $\frac{1}{a}$ such that $a(\frac{1}{a}) = (\frac{1}{a})a = 1$
	- a such that a + (-a) = (-a) + a = 0	number $\frac{1}{a}$ such that $a(\frac{1}{a}) = (\frac{1}{a})a = 1$

### Sample Problems

1. Find the dimensions:

 $\begin{bmatrix} -3 & 5\\ 4 & 1/4\\ -\pi & 0 \end{bmatrix}$  3 rows, 2 columns; Dimensions: 3 x 2

2. Two stores carry small, medium, and large sweatshirts. The table shows the inventory at the stores. Arrange the data in a matrix. Give the dimensions of the matrix.

Sweatshirt Inventory			
	Small	Medium	Large
Store A	6	21	13
Store B	16	32	28
<u>r ( 21 12</u>			

 $\begin{bmatrix} 0 & 21 & 13 \\ 16 & 32 & 28 \end{bmatrix}$  The dimensions are 2 x 3

3. Multiply the following matrix:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix}$$
$$\begin{bmatrix} 58 & 64 \\ 139 & 154 \end{bmatrix}$$

- 4. Find the inverse of the following matrix:
  - $\begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$
  - $\begin{bmatrix}
     1 & 1 & 9 \\
     0 & 1 & 9
     \end{bmatrix}$
  - .0 1 3



5. What system of equations is represented by the matrix equation?

$$\begin{bmatrix} -41 & 1 & 0 \\ 1 & 50 & 1 \\ 67 & 4 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ -29 \end{bmatrix}$$

-41x + y = 1x + 50y + z = 767x + 4y = -29



# Accelerated Pre-Calculus Unit 2: Conics

# Dear Parents,

Vocabulary

# References

Textbook Connection: Foerster PreCalculus Text: Chapter 10 In this unit, students will build on standards from previous courses, students will derive the equations of conic sections (parabolas, ellipses, and hyperbolas). Students will solve systems of a linear and quadratic equation in two variables.

# **Concepts Students will Use & Understand**

- Derive the equation of a parabola, ellipse, and hyperbola.
- Solve a linear and quadratic system in two variables.

# Helpful Links:

### GA Virtual:

http://cms.gavirtualschool. org/Shared/Math/GSEPreca lculus/Conics/index.html

Conics Introduction: http://math.about.com/libr ary/blconic.htm

### Conic Sections:

http://www.sparknotes.co m/math/precalc/conicsecti ons/

### Linear & Quadratic Systems:

http://tutorial.math.lamar. edu/Classes/Alg/NonLinear Systems.aspx

- **Cone**: A three dimensional figure with a circular or elliptical base and one vertex.
- **Coplanar**: Set of points, lines, rays, line segments, etc., that lie in the same plane.
- **Ellipse:** A curved line forming a closed loop, where the sum of the distances from two points (foci) to every point on the line is constant.
- **Focus:** one of the fixed points from which the distances to any point of a given curve, such as an ellipse or parabola, are connected by a linear relation.
- **Hyperbola:** A plane curve having two branches, formed by the intersection of a plane with both halves of a right circular cone at an angle parallel to the axis of the cone. It is the locus of points for which the difference of the distances from two given points is a constant.
- Locus of Points: A group of points that share a property. Plane: One of the basic undefined terms of geometry. A plane goes on forever in all directions (in two-dimensions) and is "flat" (i.e., it has no thickness).For further help:

http://www.teachers.ash.org.au/jeather/maths/dictionary.html http://intermath.coe.uga.edu/dictnary/homepg.asp http://www.amathsdictionaryforkids.com/

# Sample Practice Problems

1. Write the standard equation of a parabola with a vertex at the origin and with the directrix y = 4.

- 2. A parabola defined by the equations  $4x + y^2 6y = 9$  is translated 2 units up and 4 units to the left. Write the standard equation of the resulting parabola.
- 3. Mars orbits the Sun in an elliptical path whose minimum distance from the Sun is 129.5 million miles and whose maximum distance from the Sun is 154.4 million miles. The Sun represents one focus of the ellipse. Write the standard equation for the elliptical orbit of Mars around the Sun, where the center of the ellipse is at the origin.
- 4. Write the standard equation for the hyperbola with vertices at (0, -4) and (0, 4) and co-vertices at (-3, 0) and (3, 0)
- 5. Find the equations of the asymptotes and the coordinates of the vertices for the graph of  $\frac{y^2}{16} \frac{x^2}{36} = 1$

#### **Answers to Sample Practice Problems**

1. 
$$y = \frac{1}{4(-4)}x^2$$
 OR  $y = -\frac{1}{16}x^2$ 

2. 
$$x - \frac{1}{2} = -\frac{1}{4}(y - 5)^2$$

3.  $\frac{x^2}{20,149.8} + \frac{y^2}{19,994.8} = 1$ 

$$4. \quad y = \pm \sqrt{16\left(1 + \frac{x^2}{9}\right)}$$

5. The asymptotes are  $y = \pm \frac{2}{3}x$ . The vertices are (0, -4) and (0, 4)



# Accelerated PreCalculus Unit 3: Introduction to Trigonometric Functions

# References

### Dear Parents,

Textbook Connection: Foerster PreCalculus Text: Chapter 5 In this unit, students will be introduced to a basic understanding of trigonometric functions, which will be further developed upon in Unit 2. Students will apply their understanding of right triangle trig, learned in Geometry, and apply it to the unit circle. Students will also learn how to graph trig functions along with identifying and understanding characteristics specific to trig functions.

# Concepts Students will Use & Understand

- Explain what is meant by the radian measure of an angle.
- Define the trigonometric functions in terms of a point on the unit circle.
- Be able to determine the trigonometric values of one of the special real numbers by using the reference number of that real number.
- Be able to graph a trigonometric function and identify its characteristics.
- Know what is meant by the amplitude, the period, and the phase shift of a trigonometric function.
- Be able to write an equation of a trigonometric function given the characteristics of that function.
  - Be able to explain why  $(\sin t)^2 + (\cos t)^2 = 1$  is an identity, and use it to solve problems.

# Vocabulary

- Standard Position
- Initial Side
- Terminal Side
- Co-terminal Angle
- Reference Angle
- Unit Circle
- Radian
- Subtended Arc
- Sinusoidal Function
- Midline
- Amplitude
- Period
- Frequency

### For further help:

http://www.teachers.ash.org.au/jeather/maths/dictionary.html http://intermath.coe.uga.edu/dictnary/homepg.asp http://www.amathsdictionaryforkids.com/

### Helpful Links:

- □ Learn Zillion Video
- Learn Zillion Video
- □ Khan Academy
- Khan Academy Video
- Khan Academy Video
- Khan Academy
   Video

### Example 1

Convert 78° to radians. Convert  $\frac{2\pi}{3}$  radians to degrees.

### Example 2

Find the coordinates of the point where the terminal side of a  $330^{\circ}$  angle intersects the unit circle.

### Example 3

What is the  $\cos \frac{7\pi}{6}$  ?

### Example 4

Graph the function  $f(x) = 2 \sin x$  over the restricted domain  $[-2\pi, 2\pi]$ .

Then describe the period, midline, amplitude, and frequency.



### Example 5

Write an equation to describe the graphed function.





# Acc. PreCalculus Unit 4: Trigonometric Functions

# References

### Dear Parents,

Building on standards from Unit 1, students extend their study of the unit circle and trigonometric functions. Students will create inverses of trigonometric functions and use the inverse functions to solve trigonometric equations that arise in real-world problems.

# Concepts Students will Use & Understand

- Build upon understanding of the trigonometric functions
- Use special right triangles to determine the x- and y-coordinates of angles on the unit circle.
- Investigate how the symmetry of the unit circle helps to extend knowledge to angles outside of the first quadrant
- Use the symmetry of the unit circle to define sine and cosine as even and odd functions
- Investigate inverse trigonometric function
- Use trigonometric inverses to solve equations and real-world problems.

# Vocabulary

**Co-terminal Angle:** Two angles are co-terminal if they are drawn in the standard position and both have their terminal sides in the same location.

**Even Function:** A function f is even if the graph of f is symmetric with respect to the y-axis. Algebraically, f is even if and only if f(-x) = f(x) for all x in the domain of f.

**Odd Function:** A function f is odd if the graph of f is symmetric with respect to the origin. Algebraically, f is odd if and only if f(-x) = -f(x) for all x in the domain of f.

**Reference Angle:** A reference angle for angle  $\theta$  is the positive acute angle made by the terminal side of angle  $\theta$  and the x-axis.

**Special Right Triangles:** Refers to the 45-45-90 and 30-60-90 right triangles **Terminal side of angle:** The initial side of an angle lies on the x-axis. The other side, known as the terminal side, is the one that can be anywhere and defines the angle. **Unit Circle:** A unit circle is a circle that has a radius of one unit.

For further help: http://www.teachers.ash.org.au/jeather/maths/dictionary.html http://intermath.coe.uga.edu/dictnary/homepg.asp http://www.amathsdictionaryforkids.com/

# Sample Practice Problems

Textbook Connection: Foerster PreCalculus Text: Unit 2

### Helpful Links:

### GA Virtual Learning

http://cms.gavirtualschool.or g/Shared/Math/GSEPrecalcul us/TrigonometricFunctions/i ndex.html

#### Unit Circle Self-Assessment

http://www.talljerome.com/ NOLA/100528\_unitcircle.htm

#### Unit Circle Formula

http://www.mathwarehouse. com/unit-circle/graph-andformula-unit-circle.php

#### Unit Rate & Trigonometric Ratios

http://www.mathsisfun.com/ algebra/trig-interactive-unitcircle.html

Regent's Prep: Working w/ Inversion Trig Functions http://www.regentsprep.org/regen ts/math/algtrig/att8/inversetrig2.ht m



$$sinx = -\frac{1}{2}$$



# Accelerated Pre-Calculus Unit 5: Trigonometric Identities

## References

### **Dear Parents,**

In this unit students will:

- build upon their work with trigonometric identities with addition and subtraction formulas
- will look at addition and subtraction formulas geometrically
- prove addition and subtraction formulas
- use addition and subtraction formulas to solve problems

### **Concepts Students will Use & Understand**

- Demonstrate a method to prove addition or subtraction identities for sine, cosine, and tangent.
- Apply addition or subtraction identities for sine, cosine, and tangent.
- Use addition or subtraction identities to find missing values for sine, cosine and tangent functions.

### Vocabulary

- Addition Identity for Cosine: cos(x + y) = cos x cos y sin x sin y
- Addition Identity for Sine: sin(x+y) = sin x cos y + cos x sin y
- Addition Identity for Tangent:  $tan(x + y) = \frac{tan x + tan y}{1 tan x tan y}$
- **Double Angle Identity for Sine**: sin(2x) = 2 sin x cos x
- Double Angle Identity for Cosine:

$$\cos(2x) = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

- Double Angle Identity for Tangent:  $\tan 2x = \frac{2 \tan x}{1 \tan^2 x}$
- Half Angle Identity for Sine:  $\sin(\frac{x}{2}) = \pm \sqrt{\frac{1-\cos x}{2}}$
- Half Angle Identity for Cosine:  $\cos(\frac{x}{2}) = \pm \sqrt{\frac{1+\cos x}{2}}$
- Half Angle Identity for Tangent:  $\tan\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1-\cos x}{1+\cos x}} = \frac{\sin x}{1+\cos x} = \frac{1-\cos x}{\sin x}$
- **Identity**: an identity is a relation that is always true, no matter the value of the variable.
- Subtraction Identity for Cosine: cos(x y) = cos x cos y + sin x sin y
- Subtraction Identity for Sine: sin(x-y) = sin x cos y cos x sin y
- Subtraction Identity for Tangent:  $tan(x y) = \frac{tan x tan y}{1 + tan x tan y}$

### For further help:

http://www.teachers.ash.org.au/jeather/maths/dictionary.html http://intermath.coe.uga.edu/dictnary/homepg.asp http://www.amathsdictionaryforkids.com/

Textbook Connection: Precalculus with Trigonometry Text: Chapter 8

Every student will receive a text copy and access to the online textbook resource: http://my.hrw.com/

### Helpful Links:

- Addition Identities
- Khan Academy
   Addition & Double
   Angle for Sine/Cosine
- Double & Half Angles
- □ All Identities
- □ All Identities Practice

# **Sample Practice Problems**

- 1. Find the exact value of each trigonometric expression.
  - a. cos 75°
  - b. sin (-210°)
  - c.  $\tan \frac{\pi}{12}$
- 2. Find the exact value of each expression.
  - a.  $sin15^{\circ}cos75^{\circ} + cos15^{\circ}sin75^{\circ}$
  - $\frac{tan48^{\circ}+tan12^{\circ}}{1-tan48^{\circ}tan12^{\circ}}$ b.
- 3. Simplify each expression.
  - a. *sin3ycosy* + *cos3ysiny*
  - $tan 5\theta + tan \theta$ b.
  - $tan 5 \theta tan \theta 1$
- 4. Find the values of sin  $2\theta$ , cos  $2\theta$ , and tan  $2\theta$  for the given value and interval.

$$tan\theta = \sqrt{3}, (0, \frac{\pi}{2})$$

- 5. Solve each equation on the interval  $[0, 2\pi]$ .
  - a.  $\sin 2\theta = \cos \theta$
  - b.  $\tan 2\theta \tan 2\theta \tan^2 \theta = 2$
- 6. Find the exact value of each expression.
  - a. sin 67.5°
  - b.  $cos \frac{\pi}{12}$
- 7. Solve each equations on the interval  $[0, 2\pi]$ .

a. 
$$sin\frac{\theta}{2} + cos\theta = 1$$
  
b.  $tan\frac{\theta}{2} = sin\frac{\theta}{2}$ 

Answer Key

1a. $\frac{\sqrt{6}-\sqrt{2}}{4}$	1b. ½	1c. 2 – $\sqrt{3}$
2a. 1	2b. √3	
3a. sin4y	3b. <i>—tan6θ</i>	
$4.\frac{\sqrt{3}}{2};-\frac{1}{2};-\sqrt{3}$		
5a. $\frac{\pi}{6}$ , $\frac{\pi}{2}$ , $\frac{5\pi}{6}$ , $\frac{3\pi}{2}$	5b. $\frac{\pi}{4}$ , $\frac{5\pi}{4}$	
$6a.\frac{\sqrt{2+\sqrt{2}}}{2}$	6b. $\frac{\sqrt{2+\sqrt{3}}}{2}$	
7a. $0, \frac{\pi}{3}, \frac{5\pi}{3}$	7b. 0	



# <u>Acc.PreCalculus</u> Unit 6: Trigonometry of General Triangles

# References

# Dear Parents,

Textbook Connection: Foerster PreCalculus Text: Chapter 9

### Helpful Links:

#### GA Virtual Learning

http://cms.gavirtualschool.org/S hared/Math/GSEPrecalculus/Tri gOfGeneralTriangles/index.html

- http://ccssmath.org/? page\_id=2289
- http://ccssmath.org/? page\_id=2291
- http://ccssmath.org/? page\_id=2293
- http://www.shmoop.c om/common-corestandards/ccss-hs-gsrt-9.html
- http://www.shmoop.c om/common-corestandards/ccss-hs-gsrt-10.html
- http://www.shmoop.c om/common-corestandards/ccss-hs-gsrt-11.html
- https://www.engagen y.org/ccls-math/gsrt9
- https://www.engagen y.org/ccls-math/gsrt11

 https://www.opened.c om/search?category=s imilarity-righttriangles-andtrigonometry&grade= 10&grade\_group=high -schoolgeometry&standard=G .SRT.11&standard\_gro up=common-coremath Building on standards from Unit 1 and Unit 2, students will apply trigonometry to general triangles. Students will derive the trigonometric formula for the area of a triangle and prove and use the Laws of Sines and Cosines to solve problems.

# **Concepts Students will Use & Understand**

- Expand the use of trigonometric functions beyond right triangles into more general triangles.
- Develop the trigonometric formula for area of triangle.
- Use the Laws of Sines and Cosines to solve problems.

# Vocabulary

Altitude of a Triangle: The perpendicular distance between a vertex of a triangle and the side opposite that vertex. Sometimes called the height of a triangle. Also, sometimes the line segment itself is referred to as the altitude.

**Hinge Theorem:** If two sides of one triangle are congruent to two sides of another triangle, and the included angle of the first is larger than the included angle of the second, then the third side of the first triangle is longer than the third side of the second triangle. (Wikipedia) **Included Angle:** The angle between two given sides of a triangle

**Law of Cosines:** The square of the length of any side of a triangle equals t

**Law of Cosines:** The square of the length of any side of a triangle equals the sum of the squares of the lengths of the other two sides minus twice the product of the lengths of the other two sides and the cosine of the angle between them. (Swokowski, Cole)

**Law of Sines:** In any triangle, the ratio of the sine of an angle to the side opposite that angle is equal to the ratio of the sine of another angle to the side opposite that angle (Swokowski, Cole) **Oblique Triangle:** A triangle that is not a right triangle

Vertex of a Triangle: The common endpoint of the two legs that serve as the sides of a triangle

	Law	of Sines
	Case: ASA or AAS	Case: SSA
si	$\frac{a}{\ln A} = \frac{b}{\sin B} = \frac{c}{\sin C}$	$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$
-	Law of Cosin	les
Case: S.	AS	Case: SSS
$a^2 = b^2 + c^2 - $	$2bc\cos A$	$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$
$b^2 = a^2 + c^2 - $	$2ac\cos B$	$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$
$c^2 = a^2 + b^2 - b^2$	$2ab\cos C$	$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

# Sample Practice Problems

In triangle ABC find c if  $A = 36^{\circ}$ ,  $B = 101^{\circ}$ , and b = 42.7.



- b. About 40.2 units
- c. About 29.666 units
- Od. About 1.6 units

Two observers 3 miles apart and facing each other find that the angles of elevation of a balloon in the same vertical plane with themselves are 28° and 31° respectively. Find the distance from the balloon to the observer located at the 28° angle.

About	2.4	mi	les
	About	About 2.4	About 2.4 mi

- b. About 3.5 miles
- C. About 1.8 miles
- d. About 1.6 miles

Determine the number of possible solutions for traingle ABC, given  $A = 40^{\circ}$ , a = 7, and b = 9.

a. Two
 b. Three
 c. None

🔍 **d.** One

Given the following triange, find its area and express it rounded to three decimal places.



 $A = \frac{1}{2}abSinC$ 

From the figure, we know the following:

Substituting the values into the equation yields:

$$A = \frac{1}{2} 18 \cdot 12 \cdot Sin94^{\circ}$$

$$A = 107.736914 \text{ (Make sure your calculator is set to degree mode)}$$

$$A = 107.737$$



# Accelerated Pre-Calculus Unit 7: Vectors

### References

Textbook Connection: Foerster PreCalculus Text: Chapters 11-12

### Dear Parents,

Students will extend their understanding of complex numbers and their operations through graphical representations. Students will perform operations on vectors and use the operations to represent various quantities.

### **Concepts Students will Use & Understand**

- Perform arithmetic operations with complex numbers.
- Represent complex numbers & their operations on the complex plane.
- Represent & model with vector quantities.
- Perform operations on vectors.
- Perform operations on matrices & use matrices in applications.

### Vocabulary

- Vector: A mathematical object that has both magnitude and direction. Vectors can be expressed as **v**, or  $\langle a, b \rangle$ , or as a directed line segment (arrow) in the plane.
  - **Scalar:** A real number. A scalar has magnitude but not direction.
- Initial Point: The point at the "tail" of the arrow representing a vector.
   Often, the initial point is assumed to be (0, 0). This is the case in the notation (a, b).
- Terminal Point: The point at the "tip" of the arrow representing a vector.
- Magnitude of a Vector: The distance between a vector's initial and terminal points,

denoted  $||\mathbf{v}||$  or  $|\mathbf{v}|$ .  $||\mathbf{v}|| = ||\langle a, b \rangle|| = \sqrt{a^2 + b^2}$ Also called the length, norm, or absolute value of the vector.

- **Components of a Vector:** *a* and *b* in the vector  $\langle a, b \rangle$ .
- Parallel Vectors: Two or more vectors whose directions are the same or opposite.
- Equivalent Vectors: Two or more vectors that have the same direction and magnitude—i.e., whose representations are the same in the form (*a*, *b*). Note that equivalent vectors may not have the same initial and terminal points.
- **Zero Vector:** The vector  $\langle 0, 0 \rangle$ .
- **Resultant Vector:** The vector that results from adding two or more vectors.
  - **Tail-to-Head Representation:** A geometric representation of vector addition  $\mathbf{u} + \mathbf{v}$  wherein the initial point of  $\mathbf{v}$  is placed at the terminal point of  $\mathbf{u}$ . The vector beginning at the initial point of  $\mathbf{u}$  and ending at the (translated) terminal point of  $\mathbf{v}$  represents  $\mathbf{u} + \mathbf{v}$ .
- Parallelogram Representation / Parallelogram Rule: A geometric representation of vector addition u + v wherein a parallelogram is formed by placing the initial points of u and v at the same place and letting each vector represent the sides of





# Helpful Links:

### GA Virtual:

http://cms.gavirtualschool.org /Shared/Math/GSEPrecalculus /Vectors/index.html

### Teach Mathematics:

http://www.teachmathematic s.net/activities/dancingvectors.htm

### Better Explained:

http://betterexplained.com/ar ticles/vector-calculusunderstanding-the-dotproduct/

http://betterexplained.com/ar ticles/measure-any-distancewith-the-pythagoreantheorem/

### Faraday:

http://faraday.physics.utoront o.ca/PVB/Harrison/Flash/Vect ors/Add2Vectors.html

http://faraday.physics.utoront o.ca/PVB/Harrison/Flash/Vect ors/Add3Vectors.html http://faraday.physics.utoronto. ca/PVB/Harrison/Flash/Vectors/ Subtract2Vectors.html a parallelogram. The diagonal of the resulting parallelogram, starting at this shared initial point, represents  $\mathbf{u} + \mathbf{v}$ .

- Velocity: A vector whose magnitude is an object's speed (a scalar) and whose direction is the direction of the object's motion. Note that speed is a scalar—magnitude, no direction—whereas velocity tells us how fast an object is moving <u>and in what direction</u>.
- **Complex Plane:** A 2-dimensional representation of complex numbers established by a horizontal real axis and a vertical imaginary axis.
- Rectangular Form of a Complex Number: *a* + *bi*
- **cis**  $\vartheta$ : Shorthand for cos  $\vartheta + i \sin \vartheta$
- Polar Form of a Complex Number:  $r(\cos \vartheta + i \sin \vartheta) = r \operatorname{cis} \vartheta$
- Complex Conjugate of z = a + bi: z = a bi
- Modulus of a Complex Number: The distance between a number and 0 when plotted

on the complex plane:  $|z| = |a + bi| = \sqrt{zz} = \sqrt{a^2 + b^2}$ Also called absolute value or magnitude.

- Argument of *z*, arg(*z*): The angle—typically chosen in  $(-\pi, \pi]$ —formed by the positive-real axis and a segment connecting *z* to 0 in the complex plane.
- Re(z): a, the real part of the complex number z = a + bi
   Im(z): b, the coefficient of the imaginary part of the complex number z = a + bi

# **Sample Practice Problems**

- 1. Given:  $\left| \overrightarrow{u} \right| = 14, \left| \overrightarrow{\nu} \right| = 9, \& \overrightarrow{u} \parallel \overrightarrow{\nu}, find \left| \overrightarrow{u} + \overrightarrow{\nu} \right|.$ 23
- A ship leaving port sails for 100 miles in a direction 40° north of due east. Find the magnitude of the vertical and horizontal components.
   About 64 miles, about 77 miles
- 3. Find the component form of the given vector.



- What is the cross product of a = (1, 2, 3) and b = (4, 5, 6)?
   a × b = (-3, 6, -3)
- 5. Find the magnitude & direction of the sum of < 4, 5 > and < -1, 4 >. Magnitude:  $3\sqrt{10}$ ; Direction: 71.57°



# Accelerated Pre-Calculus Unit 8: Inferences & Conclusions from Data

## References

### Dear Parents,

In this unit, students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data— including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn.

### **Concepts Students will Use & Understand**

- Construct appropriate graphical displays (dot plots, histogram, and box plot) to represent sets of data values.
- Describe a distribution using shape, center and spread and use the correct measure appropriate to the distribution.
- Compare two or more different data sets using center and spread.
- Recognize data that is described well by a normal distribution.
- Estimate probabilities for normal distributions using area under the normal curve using calculators, spreadsheets and tables.
- Design a method to select a sample that represents a variable of interest from a population.
- Design simulations of random sampling and explain the outcomes in context of population and known proportions or means.
- Use sample means and proportions to estimate population values and calculate margins of error.
- Read and explain in context data from real-world reports.

### Vocabulary

- **Center:** measures of center such as median & mean.
- **Central Limit Theorem:** the mean of a sample of data will be closer to the mean of the overall population as the sample size increases.
- Confidence Interval: the probability that the interval reveals the true parameter value.
- **Empirical Rule**: in a normal distribution, the data will fall into 3 standards deviations of the mean.
- **Margin of Error:** the value in the confidence interval that shares how accurate the parameter may be.
- **Parameters:** numerical values that describe a population.
- **Sample Mean:** a statistic measuring the average of the observation in the sample.
- Sample Proportion: a statistic indicating the proportion of successes in a particular sample.
- **Sampling Distribution:** the distribution of values taken by a statistic in all possible same size and population samples.
- Sampling Variability: the fact that the value of a statistic varies in repeated random sampling.
- **Shape:** distribution described by symmetry, number of peaks, direction of skew, or uniformity.
- **Spread:** variability of the data.
- **Standard Deviation:** the square root of the variance.
- Variance: the average of the squares of the deviations of the observations from their mean.

Helpful Links: Measures of Central Tendency - video Box and Whisker Plots video Variance and Standard Deviation - video Normal Distribution video Types of Statistical Studies - video

Comparing and Sampling Populations- video Sampling Distribution video Confidence Intervals video Margin of Error - video Hypothesis Testing video

## **Sample Practice Problems**

#### Data Collection and Design

1. Adam rolled a six sided die 4 times and obtained the following results: 5, 5, 3, and 4. He computed the mean of the 4 rolls and used the result to estimate the mean of the population. Identify the parameter, sample, and statistic of interest. Calculate the identified statistic.

2. The Bennett family believes that they have a special genetic makeup because there are 5 children in the family and all of them are girls. Create a simulation of 100 families with 5 children using coins. Determine the percent of families in which all 5 children are girls and interpret your results.

3. Your classmate Jimmy presents a project to your class about carcinogens, substances that can cause cancer in living cells. When Jimmy said during his presentation that some soda ingredients may be carcinogens, you nearly spit out your root beer. Now you can't rest until you know whether soda consumption is linked to developing cancer. How would you go about investigating whether soda and cancer are linked? What are the benefits and drawbacks of using surveys, experiments, or observational studies?

### Summarizing and Interpreting Data

Two rival basketball teams each have ten players on a team. The total points scored by each player in the first five games of the season are shown below.
 Cougars: 21, 30, 8, 41, 11, 21, 26, 28, 32, 30
 Knights: 27, 15, 22, 31, 26, 22, 93, 29, 5, 20

 The coaches want to compare the points scored by a typical player on each team. What statistics should the coaches use? Create a box plot and compare those statistics. Then compare any other statistics that are appropriate so that center and spread are compared for both data sets. Identify any outliers and explain their effects.

2. Two small start-up companies are hiring. Josefina, who is interviewing for jobs at both companies, is comparing the salaries of the companies' current employees. The representative for Company A says her company's typical salary is \$42,000 per year. The company B representative says his company's typical salary is \$63,000 per year. The actual salaries, in thousands of dollars are shown below.

Company A: 21, 33, 35, 40, 42, 45, 45, 49, 160 Company B: 31, 31, 33, 38, 41, 44, 48, 238 Do the figures given by the company representatives really represent the typical salaries for each company? Based on the current employee's salaries, at which company is Josefina likely to earn more money. Explain.

### Normal Distribution Curve and its Applications

1. If a population of human body temperatures is normally distributed with a mean of  $98.2 \degree$  F and a standard deviation of  $0.7 \degree$  F, estimate the percent of temperatures between  $98.0 \degree$  F and  $99.0 \degree$  F.

### Conclusions from Data

1. In 2011, the average salary for a sample of NCAA Division 1 head football coaches was \$1.5 million per year, with a standard deviation of \$1.07 million. If there are 100 coaches in this sample, what is the standard error of the mean? What can your predict about the population mean based on the sample mean and its standard error?

2. A group of marine biologists placed tracking tags on 100 fish in Lake Erie one summer. The weight of each fish was recorded at the beginning and end of the summer. The average weight gain for all of the tagged fish was 1.2 pounds, with a standard deviation of 0.4 pound. What is the margin of error with 90% confidence for this study?



# Accelerated Pre-Calculus Unit 9: Inferences & Conclusions from Data

## References

### Dear Parents,

In this unit, students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data— including sample surveys, experiments, and simulations—and the role that randomness and careful design play in the conclusions that can be drawn.

### **Concepts Students will Use & Understand**

- Construct appropriate graphical displays (dot plots, histogram, and box plot) to represent sets of data values.
- Describe a distribution using shape, center and spread and use the correct measure appropriate to the distribution.
- Compare two or more different data sets using center and spread.
- Recognize data that is described well by a normal distribution.
- Estimate probabilities for normal distributions using area under the normal curve using calculators, spreadsheets and tables.
- Design a method to select a sample that represents a variable of interest from a population.
- Design simulations of random sampling and explain the outcomes in context of population and known proportions or means.
- Use sample means and proportions to estimate population values and calculate margins of error.
- Read and explain in context data from real-world reports.

### Vocabulary

- **Center:** measures of center such as median & mean.
- **Central Limit Theorem:** the mean of a sample of data will be closer to the mean of the overall population as the sample size increases.
- **Confidence Interval**: the probability that the interval reveals the true parameter value.
- **Empirical Rule**: in a normal distribution, the data will fall into 3 standards deviations of the mean.
- **Margin of Error:** the value in the confidence interval that shares how accurate the parameter may be.
- Parameters: numerical values that describe a population.
- **Sample Mean:** a statistic measuring the average of the observation in the sample.
- Sample Proportion: a statistic indicating the proportion of successes in a particular sample.
- **Sampling Distribution:** the distribution of values taken by a statistic in all possible same size and population samples.
- Sampling Variability: the fact that the value of a statistic varies in repeated random sampling.
- **Shape:** distribution described by symmetry, number of peaks, direction of skew, or uniformity.
- **Spread:** variability of the data.
- **Standard Deviation:** the square root of the variance.
- Variance: the average of the squares of the deviations of the observations from their mean.

Helpful Links: Measures of Central Tendency - video Box and Whisker Plots video Variance and Standard Deviation - video Normal Distribution video Types of Statistical Studies - video

Comparing and Sampling Populations- video Sampling Distribution video Confidence Intervals video Margin of Error - video Hypothesis Testing video

## **Sample Practice Problems**

#### Data Collection and Design

1. Adam rolled a six sided die 4 times and obtained the following results: 5, 5, 3, and 4. He computed the mean of the 4 rolls and used the result to estimate the mean of the population. Identify the parameter, sample, and statistic of interest. Calculate the identified statistic.

2. The Bennett family believes that they have a special genetic makeup because there are 5 children in the family and all of them are girls. Create a simulation of 100 families with 5 children using coins. Determine the percent of families in which all 5 children are girls and interpret your results.

3. Your classmate Jimmy presents a project to your class about carcinogens, substances that can cause cancer in living cells. When Jimmy said during his presentation that some soda ingredients may be carcinogens, you nearly spit out your root beer. Now you can't rest until you know whether soda consumption is linked to developing cancer. How would you go about investigating whether soda and cancer are linked? What are the benefits and drawbacks of using surveys, experiments, or observational studies?

### Summarizing and Interpreting Data

Two rival basketball teams each have ten players on a team. The total points scored by each player in the first five games of the season are shown below.
 Cougars: 21, 30, 8, 41, 11, 21, 26, 28, 32, 30
 Knights: 27, 15, 22, 31, 26, 22, 93, 29, 5, 20

 The coaches want to compare the points scored by a typical player on each team. What statistics should the coaches use? Create a box plot and compare those statistics. Then compare any other statistics that are appropriate so that center and spread are compared for both data sets. Identify any outliers and explain their effects.

2. Two small start-up companies are hiring. Josefina, who is interviewing for jobs at both companies, is comparing the salaries of the companies' current employees. The representative for Company A says her company's typical salary is \$42,000 per year. The company B representative says his company's typical salary is \$63,000 per year. The actual salaries, in thousands of dollars are shown below.

Company A: 21, 33, 35, 40, 42, 45, 45, 49, 160 Company B: 31, 31, 33, 38, 41, 44, 48, 238 Do the figures given by the company representatives really represent the typical salaries for each company? Based on the current employee's salaries, at which company is Josefina likely to earn more money. Explain.

### Normal Distribution Curve and its Applications

1. If a population of human body temperatures is normally distributed with a mean of  $98.2 \degree$  F and a standard deviation of  $0.7 \degree$  F, estimate the percent of temperatures between  $98.0 \degree$  F and  $99.0 \degree$  F.

### Conclusions from Data

1. In 2011, the average salary for a sample of NCAA Division 1 head football coaches was \$1.5 million per year, with a standard deviation of \$1.07 million. If there are 100 coaches in this sample, what is the standard error of the mean? What can your predict about the population mean based on the sample mean and its standard error?

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Accelerated Algebra I/Geometry A Teaching & Learning Framework								
		Semeste	r 1				Semester 2	
Unit 1 2 weeks	Unit 2 3 weeks	<b>Unit 3</b> 5 weeks	Unit 4 4 weeks	Unit 5 4 weeks	Unit 6 3 weeks	Unit 7 2 weeks	<b>Unit 8</b> 9 weeks	<b>Unit 9</b> 4 weeks
Relationship s Between Quantities & Expressions	Reasoning with Linear Equations & Inequalities	Modeling & Analyzing Quadratic Functions	Modeling & Analyzing Exponential Functions	Comparing & Contrasting Functions	Describing Data	Transforma- tions in the Coordinate Plane	Similarity, Congruence & Proofs	Right Triangle Trigonometry Review & Extend
MGSE9- 12.N.RN.2-3 (Properties of rational & irrational numbers) MGSE9- 12.N.Q.1-3 (Reason quantitativel y & use units to solve problems) MGSE9- 12.A.SSE.1 (Interpret expressions in context) MGSE9- 12.A.SSE.1a- b (Interpret formulas & expressions in context) MGSE9- 12.A.APR.1 (Add, subtract & multiply polynomials)	MGSE9- 12.A.CED.1-4 (Create equations that describe numbers or relationships) MGSE9- 12.A.REI.1,3,5 (Solve equations & inequalities 1- 2 variable) MGSE9- 12.A.REI.6 (Solve systems) MGSE9- 12.A.REI.0-12 (Solve equations & inequalities 2 variables) MGSE9- 12.F.BF.1 (Write a function) MGSE9- 12.F.BF.1a,2 (Arithmetic & geometric sequences) MGSE9-12.F.IF.1 (Input vs. output) MGSE9-12.F.IF.2 (Function notation) MGSE9- 12.F.IF.3-4 (Sequences & characteristics) MGSE9- 12.F.IF.3-6 (Rate of change) MGSE9- 12.F.IF.7,7a,9 (Analyze functions)	MGSE9-12.A.SSE.2 (Interpret the structure of expressions) MGSE9-12.A.SSE.3,3a- b (Equivalent forms of expressions) MGSE9-12.A.CED.1- 2,4 (Create equations that describe numbers or relationships) MGSE9-12.A.REI.1 (Justify how to solve an equation) MGSE9-12.A.REI.4,4a- b (Methods of solving quadratics) MGSE9-12.F.IF.1,3 (Write a function & build new functions) MGSE9-12.F.IF.1 (Input vs. output) MGSE9-12.F.IF.2 (Function notation) MGSE9-12.F.IF.4 (Characteristics) MGSE9-12.F.IF.5-6 (Rate of change) MGSE9-12.F.IF.3 (Write a functions) MGSE9-12.F.IF.3 (Graph functions) MGSE9-12.F.IF.8 (Write a function) MGSE9-12.F.IF.8a,9 (Compare & contrast functions)	MGSE9- 12.A.CED.1-2 (Create equations 1-2 variables) MGSE9-12.A.REI.1 (Justify how to solve an equation) MGSE9-12.F.BF.1 (Write a function) MGSE9-12.F.BF.1a,2 (Arithmetic & geometric sequences) MGSE9-12.F.BF.3 (Build new functions) MGSE9-12.F.IF.1 (Input vs. output) MGSE9-12.F.IF.2 (Function notation) MGSE9-12.F.IF.3-4 (Sequences & characteristics) MGSE9-12.F.IF.5-6 (Rate of change) MGSE9-12.F.IF.9 (Compare functions)	MGSE9-12.F.LE.1 (Linear vs exponential) MGSE9-12.F.LE.1a (Growth of functions) MGSE9- 12.F.LE.1b,c,2-3 (Changes in rate and relating to context) MGSE9-12.F.LE.5 (Interpret parameters) MGSE9-12.F.IF.3 (Build new functions) MGSE9-12.F.IF.1 (Input vs. output) MGSE9-12.F.IF.2 (Function notation) MGSE9-12.F.IF.2 (Function notation) MGSE9-12.F.IF.4 (Characteristics) MGSE9-12.F.IF.5-6 (Rate of change) MGSE9-12.F.IF.7 (Graph functions) MGSE9-12.F.IF.9 (Compare functions)	MGSE9- 12.S.ID.1 (Dot plots, histograms & box plots) MGSE9- 12.S.ID.2 (Compare data distribution) MGSE9- 12.S.ID.3 (Shape, center & spread) MGSE9- 12.S.ID.5-6 (Bivariate data) MGSE9- 12.S.ID.5-6 (Function of best fit) MGSE9- 12.S.ID.7-9 (Slope, correlation coefficient, causation & correlation)	MGSE9- 12.G.CO.1 (Precise definitions) MGSE9- 12.G.CO.2 (Coordinate plane) MGSE9- 12.G.CO.3 (Figures with rotations & reflections upon itself) MGSE9- 12.G.CO.4 (Definitions of transformatio ns) MGSE9- 12.G.CO.5 (Transformin g figures)	MGSE9-12.G.SRT.1-2 (Dilations & similarity) MGSE9-12.G.SRT.3 (AA criterion) MGSE9-12.G.SRT.4 (Prove theorems about triangles) MGSE9-12.G.SRT.5 (Congruence & similarity) MGSE9-12.G.CO.6-7 (Congruence & rigid motions) MGSE9-12.G.CO.8 (Triangle congruence) MGSE9-12.G.CO.9-11 (Prove geometric theorems) MGSE9-12.G.CO.12 (Geometric constructions) MGSE9-12.G.CO.13 (Construct regular polygons inscribed in a circle)	MGSE9-12.G.SRT.6 (Trigonometric ratios) MGSE9-12.G.SRT.7 (Sine & cosine of complementary angles) MGSE9-12.G.SRT.8 (Trigonometric ratios & Pythagorean Theorem) <u>Review</u> : All standards by differentiating for student needs <u>Extend</u> : MGSE9-12.G.C.1-2 (Similar circles; radii, chords, tangents & secants with inscribed angles)
	These units were written to build upon concepts from prior units, so later units contain tasks that depend upon the concepts addressed in earlier units. All units will include the Mathematical Practices and indicate skills to maintain							

### **Cobb County School District**

NOTE: Mathematical standards are interwoven and should be addressed throughout the year in as many different units and tasks as possible in order to stress the natural connections that exist among mathematical topics. Grades 9-12 Key: Algebra Strand: SSE = Seeing Structure in Expressions, APR = Arithmetic with Polynomial and Rational Expressions, CED = Creating Equations, REI = Reasoning with Equations and Inequalities Functions Strand: IF = Interpreting Functions, LE = Linear and Exponential Models, BF = Building Functions, TF = Trigonometric Functions

Geometry Strand: CO = Congruence, SRT = Similarity, Right Triangles, and Trigonometry, C = Circles, GPE = Expressing Geometric Properties with Equations, GMD = Geometric Measurement and Dimension, MG = Modeling with Geometry

Statistics and Probability Strand: ID = Interpreting Categorical and Quantitative Data, IC = Making Inferences and Justifying Conclusions, CP = Conditional Probability and the Rules of Probability, MD = Using Probability to Make Decisions



Accelerated Algebra I/Geometry A Teaching & Learning Framework								
Block Schedule								
Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Unit 7	Unit 8	Unit 9
Relationship s Between Quantities & Expressions	Reasoning with Linear Equations & Inequalities	Modeling & Analyzing Quadratic Functions	Modeling & Analyzing Exponential Functions	Comparing & Contrasting Functions	Describing Data	Transforma- tions in the Coordinate Plane	Similarity, Congruence & Proofs	Right Triangle Trigonometry Review & Extend
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# Acc. Geometry B/Algebra II Unit 1 Circles & Volume

# References

Textbook Connection: HMH Analytic Geometry: Unit 3 Modules 11-12

HMH Digital Textbook http://my.hrw.com

### **Helpful Links:**

### Helpful Links:

Lesson on angles: http://www.brightstor m.com/math/geometr y/circles/inscribedangles/

Lesson on segments: https://mathbitsnoteb ook.com/Geometry/Ci rcles/CRSegmentRules. html

Lesson on Constructions: http://www.math.nms u.edu/~pmorandi/Cou rseMaterials/Inscribed Triangles.html

Lesson on
 Constructions:

http://www.mathopen ref.com/consttangents .html

Lesson on Volume: http://www.mathexpr ession.com/volumeformulas.html

http://cms.gavirtualsc hool.org/Shared/Math /GSEGeometries17/GS EGeometry\_Circlesand VolumePart1\_Shared/i ndex.html

http://cms.gavirtualsc hool.org/Shared/Math /GSEGeometries17/GS EGeometry\_Circlesand VolumePart2\_Shared/i ndex.html

## **Dear Parents**

In this unit, students will explore and understand parts of a circle and their relationship to each other. Students will formalize an understanding of the development of volume formulas and use them at an application level.

# Concepts Students will Use & Understand

- Understand and apply theorems about circles.
- Construct the inscribed & circumscribed circles of a triangle and prove properties of angles for a quadrilateral inscribed in a circle.
- Construct a tangent line from a point outside a given circle to the circle.
- Find arc lengths and areas of sectors of circles.
- Explain volume formulas and use them to solve problems

# Vocabulary

**Central Angle:** an angle whose vertex is at the center of a circle **Chord:** a segment whose endpoints are on a circle **Circumcenter:** The point of intersection of the perpendicular bise

**Circumcenter:** The point of intersection of the perpendicular bisectors of the sides of a given triangle; the center of the circle circumscribed about a given triangle

**Circumscribed Circle:** a circle containing an inscribed polygon; for this unit the polygon will be a triangle and so the center of the circle will be the circumcenter of the triangle **Inscribed:** an inscribed planar shape or solid is one that is enclosed by and "fits snugly" inside

another geometric shape or solid

**Inscribed Angle:** an angle whose vertex is on the circle and whose sides contain chords of a circle **Inscribed Circle:** a circle enclosed in a polygon, where every side of the polygon is a tangent to the circle; specifically for this unit the polygon will be a triangle and so the center of the Inscribed Circle is the incenter of the triangle

Inscribed Polygon: a polygon whose vertices all lie on a circle

Point of Tangency: the point where a tangent line touches a circle

**Secant Line:** a line in the plane of a circle that intersects a circle at exactly two points **Secant Segment:** a segment that contains a chord of a circle and has exactly one endpoint outside of the circle

**Tangent Line:** a line in the plane of a circle that intersects a circle at only one point, the point of tangency

**Cavalieri's Principle:** a method, with formula given below, of finding the volume of any solid for which cross-sections by parallel planes have equal areas; this includes, but is not limited to, cylinders and prisms

Try http://intermath.coe.uga.edu/dictnary/homepg.asp or http://www.amathsdictionaryforkids.com/ for further examples.

### Sample Practice Problems



### Example 3

The two cones have the same radius. How much greater is the volume of the taller cone than the shorter cone?



Кеу

Example 1: x=4

Example 2: Find fractional portion of the circle by using 100/360. Area sector=100/360(pi)(6<sup>2</sup>). Area of the sector is 31.4 in.<sup>2</sup>

**Example 3:** The larger cone is approximately 469.14 cubic inches greater in volume.



# Acc. Algebra I/Geom. A Unit 2 Reasoning with Linear Equations & Inequalities

### Volume 1 Issue 2

# References

#### HMH Georgia Acc. Coordinate Algebra/Geom. A Textbook:

Unit 1: Modules 1; Unit 2 Modules 3-5, 9-10, 12-13; Unit 3 Modules 8-9

#### Check with you teacher for online access: my.hrw.com

#### Web Resources

- http://mathbitsnotebook.c om/Algebra1/LinearEquatio ns/LEGraphiLines.html
- http://mathbitsnotebook.c om/Algebra1/LinearEquatio ns/LEConstraintsLinearPract ice.html
- http://mathbitsnotebook.c om/Algebra1/Inequalities/I Qgraphinglinear2.html
- http://mathbitsnotebook.c om/Algebra1/Inequalities/I QGraphingPractice.html
- http://mathbitsnotebook.c om/Algebra1/Systems/SYlin earinequalities.html
- http://mathbitsnotebook.c om/Algebra1/Systems/SYGr aphIneqPractice.html
- http://mathbitsnotebook.c om/Algebra1/Functions/FN NotationEvaluation.html
- http://www.mathplay.com/slope-interceptgame.html
- http://www.webmath.com/ equline1.html
- http://www.mathplanet.co m/education/algebra-1/systems-of-linearequations-andinequalities/systems-oflinear-inequalities
- https://www.quia.com/rr/7 9715.html?AP\_rand=14742 76100

### **Dear Parents**

Below you will find a list of concepts that your child will use and understand while completing Unit 2: Reasoning with Linear Equations & Inequalities. Also included are references, vocabulary and examples that will help you assist your child at home.

### Concepts Students will Use and Understand

- Create Equations that Describe Numbers or Relationships
- Solve Equations & Inequalities
- Build a Function that Models a Relationship Between Two Quantities
  - Understand the Concept of Function & Use Function Notation
- Interpret Functions that Arise in Applications in Terms of Context
  - Analyze Functions using Different Representations

### Vocabulary

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- Arithmetic Sequence. A sequence of numbers in which the difference between any two consecutive terms is the same.
- Average Rate of Change. The change in the value of a quantity by the elapsed time. For a function, this is the change in the *y*-value divided by the change in the *x*-value for two distinct points on the graph.
- **Constant Rate of Change.** With respect to the variable x of a linear function y = f(x), the constant rate of change is the slope of its graph.
- **Continuous.** Describes a connected set of numbers, such as an interval.
- **Discrete.** A set with elements that are disconnected.
- **Domain.** The set of *x*-coordinates of the set of points on a graph; the set of *x*-coordinates of a given set of ordered pairs. The value that is the input in a function or relation.
- **End Behaviors.** The appearance of a graph as it is followed farther and farther in either direction.
- **Explicit Formula.** A formula that allows direct computation of any term for a sequence *a*1, *a*2, *a*3, ..., *an*, ...
- **Factor.** For any number *x*, the numbers that can be evenly divided into *x* are called factors of *x*. For example, the number 20 has the factors 1, 2, 4, 5, 10, and 20.
- Interval Notation. A notation representing an interval as a pair of numbers. The numbers are the endpoints of the interval. Parentheses and/or brackets are used to show whether the endpoints are excluded or included.
- Linear Function. A function with a constant rate of change and a straight line graph.
- Linear Model. A linear function representing real-world phenomena. The model also represents patterns found in graphs and/or data.
- **Parameter.** The independent variable or variables in a system of equations with more than one dependent variable.
- Range. The set of all possible outputs of a function.
- **Recursive Formula.** A formula that requires the computation of all previous terms to find the value of *an*.

- http://www.purplemath.co m/modules/fcnops.htm
- Slope. The ratio of the vertical and horizontal changes between two points on a surface or a line.
- X-intercept. The point where a line meets or crosses the x-axis
- Y-intercept. The point where a line meets or crosses the y-axis

### **Algebra 1 Unit 2 Practice Problems**

### Formulas

y = mx + b

Slope-Intercept:

#### Example 1

The sum of two consecutive integers is less than 83. Find the pair of integers with the greatest sum.

#### Example 2

**Arithmetic Sequence:** 

 $A_n = a_1 + (n-1)d$ 

Pablo and his family are driving to California for vacation. The trip is 1,505 miles and they drive at an average speed of 59 mph. Which equation would give the number of miles remaining until they reach their destination, M, in terms of h, the number of hours they have driven?

A. M = 59 + 1,505hB. M = 1,505 - 59hC. M = 1,505 + 59hD. M = 59 - 1,505h

#### Example 3

Britany is leaving for an 800 mile road trip. Her plan is not to make any stops until she has 590 miles, or less, left of the drive. She is averaging 70 miles per hour. If *x* represents the number of hours driving, which of the following inequalities symbolizes this situation?

A. 590 - 70x > 800	B. 800 - 70x < 590
C. 590 - 70x < 800	D. 800 - 70x > 590

#### Example 4

What is the next term in this sequence? 4, 10, 16, ...

#### Example 5

Generate ordered pairs for the function y = x + 3 for x = -2, -1, 0, 1, and state the domain and range.

#### **Answer Key**

#### Example 1

Define a Variable: x = the first consecutive number, so x +1 = the second consecutive number Equation: x + x + 1 < 83 2x < 82 x < 41

The numbers are 40 and 41 Check: 40 + 41 < 83 81 < 83

#### Example 2

B. M = 1,505 – 59h Example 3 B. 800 - 70x < 590 Example 4 22 Example 5 (-2,1), (-1,2), (0,3), (1,4), (2,5) Domain: {-2,-1,0,1,2} Range: {1,2,3,4,5})



# Acc. Algebra I/Geom. A Unit 3: Modeling & Analyzing Quadratic Functions

### Volume 1 Issue 3

# References

HMH Georgia Analytic Geometry Text: Unit 5: Modules 14-16

Check with you teacher for online and print access:

> Online website: my.hrw.com

#### Web Resources

- http://www.purplemath
   .com/modules/quadfor
   m.htm (quadratic
   equation)
- http://www.purplemath .com/modules/solvquad
   .htm (solving quadratic equations)
- http://www.purplemath
   .com/modules/grphqua
   d2.htm (vertex form)
- http://www.analyzemat
   h.com/quadratics/quadr
   atics.htm (standard
   form)
- http://www.purplemath .com/modules/ineqqua d.htm (quadratic inequalities)

### **Dear Parents**

Below you will find a list of concepts that your child will use and understand while completing Unit 3: Modeling & Analyzing Quadratic Functions. Also included are references, vocabulary and examples that will help you assist your child at home.

## Concepts Students will Use and Understand

Students will analyze quadratic functions in the forms  $f(x) = ax^2 + bx + c$  and  $f(x) = a(x - h)^2 + k$ 

- Convert between standard and vertex form
  - Graph quadratic functions as transformations of  $f(x) = x^2$
- Investigate & explain characteristics of quadratic functions
- Explore quadratic sequences recursively and explicitly
- Students will solve quadratic equations and inequalities in one variable.
- Solve equations graphically & with technology
- Find real & complex solutions of equations by factoring, taking square roots, and applying the quadratic formula
- Analyze roots using technology and the discriminant
- Solve quadratic inequalities both graphically and algebraically
- Describe the solutions using linear inequalities and interval notation

# Vocabulary and Theorems

- **Complete factorization over the integers.** Writing a polynomial as a product of polynomials so that none of the factors is the number 1, there is at most one factor of degree zero, each polynomial factor has degree less than or equal to the degree of the product polynomial, each polynomial factor has all integer coefficients, and none of the factor polynomial can written as such a product.
- **Completing the square**. Completing the Square is the process of converting a quadratic equation into a perfect square trinomial by adding or subtracting terms on both sides.
- **Difference of two squares**. A squared (multiplied by itself) number subtracted from another squared number. It refers to the identity  $a^2 b^2 = (a + b)(a b)$  in elementary algebra.
- **Discriminant of a quadratic equation.** The discriminant of a quadratic equation of the form

 $ax^2 + bx + c = 0$ ,  $a \neq 0$ , is the number  $b^2 - 4ac$ .

- Horizontal shift. A rigid transformation of a graph in a horizontal direction, either left or right.
- Perfect square trinomial. A trinomial that factors into two identical binomial factors.
- Quadratic equation. An equation of degree 2, which has at most two solutions.
- Quadratic function. A function of degree 2 which has a graph that "turns around" once, resembling an umbrella–like curve that faces either right–side up or upside down. This graph is called a parabola.
- **Root.** The *x*-values where the function has a value of zero.
- Standard form of a quadratic function.  $ax^2 + bx + c$

parabola is opening up or down, or in terms of x if the parabola is opening left or right.

······

Vertex form of a quadratic function. A formula for a quadratic equation of the form f
 (x) = a (x - h)<sup>2</sup> + k, where a is a nonzero constant and the vertex of the graph is the point (h, k).

#### Theorems

For 
$$h = \frac{-b}{2a}$$
 and  $k = f\left(\frac{-b}{2a}\right)$ ,  $f(x) = a(x-h)^2 + k$  is the same function as  $f(x) = ax^2 + bx + c$ .

The graph of any quadratic function can be obtained from transformations of the graph of the basic function  $f(x) = x^2$ .

**Quadratic formula**: The solution(s) of the quadratic equation of the form  $ax^2 + bx + c = 0$ , where a, b, and c are real numbers with  $a \neq 0$ , is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The discriminant of a quadratic equation is positive, zero, or negative if and only if the equation has two real solutions, one real solution, or two complex conjugate number solutions respectively.

### **Algebra 1 Unit 3 Practice Problems**

Formulas Example 1 What happens to the graph of  $y = x^2$  when you multiply  $x^2$  by 3? **Quadratic Equations:** Standard Form: Example 2 Find the zeros of (2x + 3)(3x + 4) = 0.  $y = ax^2 + bx + c$ Example 3 Factor 6x<sup>2</sup> +7x -20 Vertex Form: Example 4  $y = a(x - h)^{2} + k$ Solve the quadratic equation:  $2x^2 + 3x - 54 = 0$ . Example 5 **Quadratic Formula:** Find the y-intercept of  $f(x) = x^2 - 4x + 9$ .  $-b \pm \sqrt{b^2 - 4ac}$ Example 6 Find the discriminate of  $3x^2 + 15x = 12$ **Answer Key** Example 1 It causes a vertical stretch. (see graph to right) 20Example 2 x = -3/2 or x = -4/3Example 3 15 (2x + 5)(3x - 4)Example 4 (2x-9)(x+6) = 010x = 4.5 or x= -6  $y = x^2$ Example 5  $f(0) = 0^2 - 4(0) + 9$ y-intercept = 9 Example 6 b<sup>2</sup>-4ac = (15)<sup>2</sup>-4(-36) = 369



# Acc. Algebra 1/Geom. A Unit 4: Modeling & Analyzing Exponential Functions

#### Volume 1 Issue 4

# References

### HMH Georgia Coordinate Algebra Text:

Unit 3: Modules 12-13

# Check with you teacher for online and print access:

Online website: my.hrw.com

### Web Resources

• Graphing Exponential Functions http://cms.gavirtualschool.org/Shar ed/Math/GSEAlgl16/GSEAlgl\_Propof ExpandGraphExpFunc\_Shared/GSEAl gl\_PropofExpandGraphExpFunc\_Sha red4.html

#### • Characteristic of Exponential Functions

http://cms.gavirtualschool.org/Shar ed/Math/GSEAlgI16/GSEAlgI\_Propof ExpandGraphExpFunc\_Shared/GSEAl gI\_PropofExpandGraphExpFunc\_Sha red7.html

#### • Exponential Functions

http://www.purplemath.com/modul es/expofcns.htm

• Exponential functions

https://mathbitsnotebook.com/Alge bra1/FunctionGraphs/FNGTypeExpo nential.html

### Exponential functions

http://www.mathsisfun.com/sets/function-exponential.html

• Geometric sequences http://mathbitsnotebook.com/Algeb ra1/Functions/FNSequences.html

• Geometric sequences http://www.purplemath.com/modul es/series3.htm

#### Geometric sequences

http://www.basicmathematics.com/geometricsequence.html

### **Dear Parents**

Below you will find a list of concepts that your child will use and understand while completing Unit 4: Modeling & Analyzing Exponential Functions. Also included are references, vocabulary and examples that will help you assist your child at home.

### **Concepts Students will Use and Understand**

- Analyze & interpret exponential functions in real-world applications.
- Build on and informally extend understanding of integer exponents to consider exponential functions.
- Use function notation.
- Interpret expressions for functions in terms of the situation they model.
- Analyze exponential functions and model how different representations may be used based on the situation presented.
- Build a function to model a relationship between two quantities.
- Recognize geometric sequences as exponential functions.
- Create new functions from existing functions.
- Construct and compare exponential models and solve problems.
- Reinforce their previous understanding of characteristics of graphs and investigate key features of exponential graphs.
- Investigate a multiplicative change in exponential functions.
- Create and solve exponential equations.
- Apply related linear equations solution techniques and the laws of exponents to the creation and solution of simple exponential equations.

### Vocabulary

- **Explicit Expression.** A formula that allows direct computation of any term for a sequence *a*<sub>1</sub>, *a*<sub>2</sub>, *a*<sub>3</sub>, ..., *a*<sub>n</sub>, ....
- **Exponential Function.** A nonlinear function in which the independent value is an exponent in the function, as in  $y = ab^x$ .
- **Exponential Model.** An exponential function representing real-world phenomena. The model also represents patterns found in graphs and/or data.
- **Geometric Sequence.** A sequence of numbers in which the ratio between any two consecutive terms is the same. In other words, you multiply by the same number each time to get the next term in the sequence. This fixed number is called the common ratio for the sequence.
- **Recursive Formula.** A formula that requires the computation of all previous terms to find the value of *a<sub>n</sub>*.

Exponential Growth & Decay	Compound Interest
Growth:     Decay: $y = a(1+r)^x$ $y = a(1-r)^x$ $a$ = initial amount before measuring growth/decay $r$ = growth/decay rate (often a percent) $x$ = number of time intervals that have passed	$\begin{split} A &= P\left(1+\frac{r}{n}\right)^{nt}\\ \text{Where,}\\ &= P = \text{principal amount (initial investment)}\\ &= r = \text{annual nominal interest rate (as a decimal)}\\ &= n = \text{number of the the interest is compounded per yeal}\\ &= t = \text{number of years} \end{split}$
Geometric Sequence: Recursive $a_n = a_{n-1}r$	Geometric Sequence: Explicit $a_n = a_1 r^{n-1}$

## **Practice Problems**

### Formulas

#### **Exponential Growth/Decay:**



#### **Compound Interest:**

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$
  
Where,  
• P = principal amount (initial investment)  
• r = annual nominal interest rate (as a decimal)  
• n = number of times the interest is compounded per year  
• t = number of years

#### **Geometric Sequences:**

#### Recursive

$$a_n = a_{n-1}r$$

#### Explicit



1.) Tell whether each set of ordered pairs satisfies an exponential function. Explain your answer.

 $\{(-1, 1), (0, 0), (1, 1), (2, 4)\}$ 

Yes; as the x-values change by a constant amount, the y-values are multiplied by a constant amount.

2.) Choose several values of x and generate ordered pairs. Then use the ordered pairs to graph the function:  $y = 0.2(5^x)$ 



x	у
<mark>-1</mark>	<mark>.04</mark>
<mark>0</mark>	<mark>.2</mark>
<mark>1</mark>	<mark>1</mark>
2	<mark>5</mark>

3.) In the definition of an exponential function, the value of *b* cannot be 1, and the value of *a* cannot be 0. Why?

If the value of *b* were 1, the function would be constant. If the value of *a* were 0, the function would be the constant function of y = 0.

#### 4.) Technology Application:

Moore's law states that the maximum number of transistors that can fit on a silicon chip doubles every two years. The function  $f(x) = 42(1.41)^x$  models the number of transistors, in millions, that can fit on a chip, where x is the number of years since 2000. Using this model, in what year can a chip hold 1 billion transistors?

#### About 2009

5.) The population of a town is decreasing at a rate of 3% per year. In 2000, there were 1700 people. Write an exponential decay function to model this situation. Then find the population in 2012.

 $\frac{y = 1700(0.97)^t}{\text{Population: } 1180 \text{ people}}$ 



# Acc. Algebra I/Geom. A Unit 5: Comparing & Contrasting Functions

#### Volume 1 Issue 5

# References

### HMH Georgia Advanced Algebra Text:

Unit 3: Module 13

# Check with you teacher for online and print access:

Online website: my.hrw.com

### Web Resources

### • Rate of Change

http://www.nms.org/Portals/0/D ocs/FreeLessons/Fill%20lt%20Up ,%20Please%20-%20Part%20Up.pdf

%20Part%20III.pdf

• Distinguishing between Linear & Exponential Functions

https://learnzillion.com/lesson\_p lans/6663/

• Comparing Graphs of Linear, Quadratic, & Exponential Functions

http://www.virtualnerd.com/alg ebra-1/quadratic-equationsfunctions/linear-exponentialcomparison/linear-exponentialcomparison-graphingexamples/determine-functiontype-from-graph

### **Dear Parents**

Below you will find a list of concepts that your child will use and understand while completing Unit 5: Comparing & Contrasting Functions. Also included are references, vocabulary and examples that will help you assist your child at home.

### **Concepts Students will Use and Understand**

- Deepen their understanding of linear, quadratic, and exponential functions as they compare and contrast the three types of functions.
- Understand the parameters of each type of function in contextual situations.
- Interpret linear, quadratic, and exponential functions that arise in applications in terms of the context.
- Analyze linear, quadratic, and exponential functions and model how different representations may be used based on the situation presented.
- Construct and compare characteristics of linear, quadratic, and exponential models and solve problems.
- Distinguish between linear, quadratic, and exponential functions graphically, using tables, and in context.
- Recognize that exponential and quadratic functions have a variable rate of change while linear functions have a constant rate of change.
- Distinguish between additive and multiplicative change and construct and interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.
- Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

### Vocabulary

All vocabulary is repeated from units 2-4.



## **Unit 5 Practice Problems**

- Answers
- 1.) <mark>Exponential</mark>



2.) Linear: Constant Quadratic: Variable Exponential: Variable (Average rate of change will eventually be greater than quadratic or linear rates of change.)

3.) Quadratic Function

4.) Plan A: y = 50x + 500Plan B:  $y = 500(1.05)^{x}$ 



More homes will be built under plan A up to the end of the 26<sup>th</sup> year. After that, more homes will be built under plan B, and plan B results in more homes than plan A by ever-increasing amounts each year. 1.) Graph the set of data. Which kind of model best describes the data?  $\{(-1, 4), (-2, 0.8), (0, 20), (1, 100), (-3, 0.16)\}$ 

2.) Describe the rate of change for linear, quadratic, & exponential functions.

3.) Determine which function model the data in the table represents:

Height of Bridge Suspension Cables		
Cable's Distance	Cable's Height	
from Tower (ft)	(ft)	
0	400	
100	256	
200	144	
300	64	

4.) A town home has approximately 500 homes. The town council is considering plans for future development. Plan A calls for an increase of 50 homes per year. Plan B calls for a 5% increase each year. Compare the plans.



# Acc. Algebra I/Geom. A Unit 6: Describing Data

#### Volume 1 Issue 6

# References

HMH Georgia Coordinate Algebra Text: Unit 4: Modules 14 & 15 HMH Georgia Analytic Geometry Text: Unit 5: Module 15.3 (quadratic regression)

#### Check with you teacher for online and print access:

Online website: my.hrw.com

#### Web Resources

#### GA Virtual: Interpreting and Representing Two Variable Data

http://cms.gavirtualschool.org/S hared/Math/GSEAlgI16/GSEAlgI\_ IntandRepTwoVarData\_Shared/i ndex.html

• Khan Academy https://www.khanacademy.org/mat h/probability/regression

 Correlation Coefficient http://mathbits.com/MathBits/TI Section/Statistics2/correlation.ht m

• Two-way Frequency Tables

https://mathbitsnotebook.com/Alge bra1/StatisticsReg/ST2TwoWayTable .html

• Shapes of Distributions

http://www.mathbitsnotebook.c om/Algebra1/StatisticsData/STSh apes.html

### **Dear Parents**

Below you will find a list of concepts that your child will use and understand while completing Unit 6: Describing Data. Also included are references, vocabulary and examples that will help you assist your child at home.

### **Concepts Students will Use and Understand**

- Know how to compute the mean, median, interquartile range, and mean absolute deviation by hand in simple cases and using technology with larger data sets.
- Find the lower extreme (minimum), upper extreme (maximum), and quartiles.
- Use and interpret shape, center, and spread of data.
- Create a graphical representation of a data set.
- Summarize data in two-way frequency table.
- Represent data in a scatter plot and describe how the variables are related.
- Interpret the slope & y-intercept of a line from any representation.
- Find linear, quadratic, and exponential regressions.
- Compute and interpret the correlation coefficient.
- Understand the meaning of correlation and causation.

### Vocabulary

- **Bivariate data.** Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.
- **Conditional Frequencies.** The relative frequencies in the body of a two-way frequency table.
- **Correlation Coefficient.** A measure of the strength of the linear relationship between two variables that is defined in terms of the (sample) covariance of the variables divided by their (sample) standard deviations.
- Joint Frequencies. Entries in the body of a two-way frequency table.
- **Marginal Frequencies.** Entries in the "Total" row and "Total" column of a two-way frequency table.
- Mean Absolute Deviation. A measure of variation in numerical data by adding the distance between each data point and the mean, then dividing by the number of values
- **Shape**. The shape of a distribution is described by symmetry, number of peaks, direction of skew, or uniformity.
- **Symmetry**. A symmetric distribution can be divided at the center so that each half is a mirror image of the other.
- **Number of Peaks**. Distributions can have few or many peaks. Distributions with one clear peak are called unimodal and distributions with two clear peaks are called bimodal. Unimodal distributions are sometimes called bell-shaped.
- **Direction of Skew**. Some distributions have many more observations on one side of graph than the other. Distributions with a tail on the right toward the higher values are said to be skewed right; and distributions with a tail on the left toward the lower values are said to be skewed left.
- **Uniformity-** When observations in a set of data are equally spread across the range of the distribution, the distribution is called uniform distribution. A uniform distribution has no clear peaks.
- Trend. A change (positive, negative or constant) in data values over time.

## **Practice Problems**



 When you look at the shape of the data, if the "long tail" is on the left=skewed left, if it is on the right=skewed right, and if it is evenly distributed it is symmetric.



3.) The correlation coefficient is approximately .999. This means the line of best fit is extremely accurate because the coefficient is so close to 1.

4.) table in yellow at bottom is the key
a. 36%, 19%, 45%, 54%, 46%
b. 25%, 9%, 20%(top), 11%, 10%,
25%(bottom)
c. 54%
d. 36%

e. <mark>69.4%</mark>

1.) Find the linear regression of the following data:

Fahrenheit degrees (°F)	Celsius degrees (°C)
32	0
68	20
86	30
122	50
158	70
194	90
212	100

2.) Explain when data is skewed left, right, or symmetric.

3.) Using technology, determine the correlation coefficient. Interpret its meaning. (0,20) (1,40) (2,75) (3,150) (4, 297) (5,510)

4.) Construct a frequency table from the following information: A survey of 200 9th and 10th graders was given to determine what their favorite subject was. 72 said Math (50 which were freshmen), 38 said Social Studies (20 which were sophomores), and 40 freshmen and 50 sophomores said PE was their favorite.

Based on your tables above, answer the following questions:

- a. What are the marginal relative frequencies?
- b. What are the joint relative frequencies?
- c. What is the probability that a student surveyed is a freshman?
- d. What is the probability that a student surveyed likes Math?
- e. If a student likes Math, what is the probability that they are a freshman?

	Math	SS	PE	Total
9th	50	18	40	108
10th	22	20	50	92
Total	72	38	90	200



# Acc. Algebra I/Geometry A Unit 7 Transformations in the Coordinate Plane

### References Textbook Connection

HMH Acc. Coordinate Algebra I/Geom. A Unit 5: Modules 16-17

Online Textbook Access: http://my.hrw.com

Ask your teacher for log in directions.

### **Helpful Links:**

http://www.mathwarehou se.com/transformations/

http://www.gradeamathh elp.com/transformationgeometry.html

http://www.onlinemathle arning.com/transformatio n-in-geometry.html

http://mathbitsnotebook. com/Geometry/Transform ations/TRRigidTransformat ions.html

http://cms.gavirtualschool .org/Shared/Math/GSECoo rdinateAlgebra/Transform ations/index.html

# Dear Parents

In this unit students will take a closer look at translations, rotations, and reflections on the coordinate plane. Students will develop a better understanding of transformations by using a variety of tools.

# Concepts Students will Use & Understand

- Know precise definitions of geometric figures
- Represent transformations in the plane and describe as functions
- Describe the rotations/reflections given a rectangle, parallelogram, trapezoid or regular polygon that carry it onto itself
- Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines and line segments
- Given a geometric figure and a rotation, reflection or translation, draw the transformed figurespecify a sequence of transformations that will carry a given figure onto another.

# Vocabulary

- Angle: A figure created by two distinct rays that share a common endpoint (also known as a vertex). ∠ABC or ∠B or ∠CBA indicate the same angle with vertex B.
- Angle of Rotation: The amount of rotation (in degrees) of a figure about a fixed point such as the origin.
- Bisector: A point, line or line segment that divides a segment or angle into two equal parts.
- Circle: The set of all points equidistant from a point in a plane.
- Congruent: Having the same size, shape and measure. ∠A ≅ ∠B indicates that angle A is congruent to angle B.
- Corresponding angles: Angles that have the same relative position in geometric figures.
- Corresponding sides: Sides that have the same relative position in geometric figures.
- Endpoint: The point at each end of a line segment or at the beginning of a ray.
- Image: The result of a transformation.
- Intersection: The point at which two or more lines intersect or cross.
- **Isometry:** a distance preserving map of a geometric figure to another location using a reflection, rotation or translation.  $M \rightarrow M'$  indicates an isometry of the figure M to a new location M'. M and M' remain congruent.
- Line: One of the undefined terms of geometry that represents an infinite set of points with no thickness and its length continues in two opposite directions indefinitely.  $\overrightarrow{AB}$  indicates a line that passes through points A and B.
- Line segment: A part of a line between two points on the line.  $\overline{AB}$  indicates the line segment between points A and B.
- Parallel lines: Two lines are parallel if they lie in the same plane and do not intersect.  $\overrightarrow{AB} \parallel \overrightarrow{CD}$  indicates that line AB is parallel to line CD.
- **Perpendicular lines:** Two lines are perpendicular if they intersect to form right angles.  $\overrightarrow{AB} \perp \overrightarrow{CD}$  indicates that line AB is perpendicular to line CD.
- **Point:** One of the basic undefined terms of geometry that represents a location. A dot is used to symbolize it and it is thought of as having no length, width or thickness.
- **Pre-image:** A figure before a transformation has taken place.
- **Ray:** A part of a line that begins at a point and continues forever in one direction.  $\overrightarrow{AB}$  indicates a ray that begins at point A and continues in the direction of point B indefinitely.
- Reflection: A transformation of a figure that creates a mirror image, "flips," over a line.
- Reflection Line (or line of reflection): A line that acts as a mirror so that corresponding points are the same distance from the mirror.
- Rotation: A transformation that turns a figure about a fixed point through a given angle and a given direction, such as 90° clockwise.
- Segment: See line segment.
- **Transformation:** The mapping, or movement, of all points of a figure in a plane according to a common operation, such as translation, reflection or rotation.
- Translation: A transformation that slides each point of a figure the same distance in the same direction.
- Vertex: The location at which two lines, line segments or rays intersect.

Try http://intermath.coe.uga.edu/dictnary/homepg.asp or http://www.amathsdictionaryforkids.com/ for further examples.

### Example 1

```
Skill-based Task
```

Which of the following preserves distance and which does not?  $(x, y) \rightarrow (x + 1, y + 2)$  $(x, y) \rightarrow (x^2, y + 1)$ 

### Example 2

y = -x.

Translation (x, y)  $\rightarrow$  (x + 4, y - 2). Rotation 180° about the origin. Reflection about the line



#### Example 3

Identify the coordinates of point (-7, -6) under the rotation of 90° clockwise about the origin? a. (7, 6)

b. (6, -7) c. (-6, 7) d. (-7, 6)

#### Кеу

Example 1

The first one preserves distance since it is a translation with adding and subtracting. The second one has a quadratic applied, so the distance is not constant.

Example 2



Black to blue to yellow to red.

Example 3

С



## Acc. Algebra I/Geometry A Unit 8 Similarity, Congruence, and Proofs

#### References

#### **Textbook Connection:**

Holt McDougal Acc. Coordinate Algebra/Analytic Geometry A: Unit 1

#### **Helpful Links:**

http://mathbitsnote book.com/Geometr y/Transformations/T **RTransformationDila** tions.html

- □ Dilations: http://mathbitsnote book.com/Geometr y/Similarity/SMdilati
- □ Similarity: http://mathbitsnote book.com/Geometr y/Similarity/SMSimil ar.html
- □ Proving Similar Triangles:

http://mathbitsnote book.com/Geometr y/Similarity/SMProo fs.html

### □ Triangle Theorems:

http://mathbitsnote book.com/Geometr y/CongruentTriangle s/CTtriangleMethod

#### □ Ratio Segments:

http://www.walterfendt.de/m14e/pro psegments.htm

- □ Congruent Triangles: http://www.analyze math.com/Geometr y/congruent\_triangl es.html
- □ Points of Concurrency: http://www.online

### **Dear Parents**

In this unit, students will understand similarity in terms of similarity transformations, prove theorems involving similarity, understand congruence in terms of rigid motions, prove geometric theorems, and make geometric constructions.

### **Concepts Students will Use & Understand**

- Understand similarity in terms of similarity transformations (dilations). •
- Prove theorems involving similarity (proportionality & Pythagorean Theorem)
- Understand congruence in terms of rigid motion (ASA, SAS, SSS) •
- Prove geometric theorems (special angles, triangles, parallelograms) •
- Make geometric constructions (copy segment/angle; bisect segment/angle; construct perpendicular/parallel lines; equilateral triangle, square and a regular hexagon inscribed in a circle

#### Vocabulary

•

- Adjacent Angles: Angles in the same plane that have a common vertex and a common side, but no common interior points.
- Alternate Exterior Angles: Alternate exterior angles are pairs of angles formed when a third line (a transversal) crosses two other lines. These angles are on opposite sides of the transversal and are outside the other two lines. When the two other lines are parallel, the alternate exterior angles are equal.
- Alternate Interior Angles: Alternate interior angles are pairs of angles formed when a third • line (a transversal) crosses two other lines. These angles are on opposite sides of the transversal and are in between the other two lines. When the two other lines are parallel, the alternate interior angles are equal.
- **Bisector:** A bisector divides a segment or angle into two equal parts. •
- **Centroid:** The point of concurrency of the medians of a triangle. •
- Circumcenter: The point of concurrency of the perpendicular bisectors of the sides of a triangle.
- Coincidental: Two equivalent linear equations overlap when graphed. ٠
- **Dilation**: Transformation that changes the size of a figure, but not the shape.
- Equiangular: The property of a polygon whose angles are all congruent. •
- Equilateral: The property of a polygon whose sides are all congruent.
- Exterior Angle of a Polygon: an angle that forms a linear pair with one of the angles of the • polygon.
- **Incenter:** The point of concurrency of the bisectors of the angles of a triangle.
- Intersecting Lines: Two lines in a plane that cross each other. Unless two lines are • coincidental, parallel, or skew, they will intersect at one point.
- Linear Pair: Adjacent, supplementary angles. Excluding their common side, a linear pair • forms a straight line.

#### $180^{\circ}(n-2)$ Measure of each Interior Angle of a Regular n-gon:

- **Orthocenter:** The point of concurrency of the altitudes of a triangle. ٠
- Plane: One of the basic undefined terms of geometry. Traditionally thought of as going on forever in all directions (in two-dimensions) and is flat (i.e., it has no thickness).
- Reflection: A transformation that "flips" a figure over a line of reflection •

### □ Dilations:

mathlearning.com/c oncurrncypoints.html

Isosceles Triangles: http://mathbitsnote book.com/Geometr y/SegmentsAnglesTr iangles/SATIsosceles .html

Constructions: http://www.mathsis fun.com/geometry/ constructions.html

- **Reflection Line:** A line that is the perpendicular bisector of the segment with endpoints at a pre-image point and the image of that point after a reflection.
- **Regular Polygon:** A polygon that is both equilateral and equiangular.
- **Remote Interior Angles of a Triangle:** the two angles non-adjacent to the exterior angle.
  - **Rotation:** A transformation that turns a figure about a fixed point through a given angle and a given direction.
  - Same-Side Interior Angles: Pairs of angles formed when a third line (a transversal) crosses two other lines. These angles are on the same side of the transversal and are between the other two lines. When the two other lines are parallel, same-side interior angles are supplementary.
- Same-Side Exterior Angles: Pairs of angles formed when a third line (a transversal) crosses two other lines. These angles are on the same side of the transversal and are outside the other two lines. When the two other lines are parallel, same-side exterior angles are supplementary.
- Scale Factor: The ratio of any two corresponding lengths of the sides of two similar figures.
- Similar Figures: Figures that have the same shape but not necessarily the same size.
- **Skew Lines:** Two lines that do not lie in the same plane (therefore, they cannot be parallel or intersect).
- Sum of the Measures of the Interior Angles of a Convex Polygon:  $180^{\circ}(n-2)$ .
- **Transformation**: The mapping, or movement, of all the points of a figure in a plane according to a common operation.
- **Translation:** A transformation that "slides" each point of a figure the same distance in the same direction
- Transversal: A line that crosses two or more lines.
- Vertical Angles: Two nonadjacent angles formed by intersecting lines or segments. Also called opposite angles.

Try http://intermath.coe.uga.edu/dictnary/homepg.asp or http://www.amathsdictionaryforkids.com/ for further examples.



### Example 2

What theorem would prove these 2 triangles congruent?



### Example 3

Construct a regular hexagon inside of a circle.

Example 1: Yes these 2 triangles are similar because their sides are proportional. The scale factor of the dilation from the smaller triangle to the larger triangle is 2.

#### <u>Example 2:</u> ASA because $\overline{KT} \cong \overline{TK}$ and $\angle IKT \cong \angle EKT$ ; $\angle ITK \cong \angle ETK$ Example 3:

We start with a line segment AF. This will become one side of the hexagon. Because we are constructing a regular hexagon, the other five sides will have this length also.	۸	<ol> <li>Move the compass on to A and draw an arc across the circle. This is the next vertex of the hexagon.</li> </ol>	
2. From points A and F, draw two arcs so that	,,	<ol><li>Move the compass to this arc and draw an arc across the circle to create the next vertex.</li></ol>	
they intersect. Mark this as point O. This is the center of the hexagon's circumcircle.	A		
<ol> <li>Move the compass to O and draw a circle.</li> <li>This is the hexagon's circumcircle - the circle that passes through all six vertices</li> </ol>		<ol> <li>Continue in this way until you have all six vertices. (Four new ones plus the points A and F you started with.)</li> </ol>	
		<ol> <li>Draw a line between each successive pairs of vertices.</li> </ol>	
		<ol> <li>Done. These lines form a regular hexagon where each side is equal in length to AF.</li> </ol>	A

### Key



# Acc. Algebra I/Geometry A Unit 9: Right Triangle Trigonometry

#### Dear Parents,

Below is information regarding Unit 9, Right Triangle Trigonometry.

#### In this unit students will:

- explore the relationships that exist between sides and angles of right triangles
- build upon their previous knowledge of similar triangles and of the Pythagorean Theorem to determine the side length ratios in special right triangles
- understand the conceptual basis for the functional ratios sine and cosine
- explore how the values of these trigonometric functions relate in complementary angles
- to use trigonometric ratios to solve problems
- develop the skills and understanding needed for the study of many technical areas
- build a strong foundation for future study of trigonometric functions of real numbers



Right Triangle Trigonometry Vocabulary Terms/Properties
Complementary Angles: two angles whose sum is 90°
sine of $h = sin(h) = \frac{length of opposite side}{length of opposite side}$
length of the hypotenuse
length of adjacent side
$\frac{\cos(\theta)}{\theta} = \cos(\theta) = \frac{1}{10000000000000000000000000000000000$
tangent of $\theta = tan(\theta) = \frac{1}{2} ength of opposite side}{1}$
length of adjacent side
Properties, theorems & corollaries:
1) 30°-60°-90° triangles pattern: hypotenuse, shorter leg, longer leg= 2a, a, a $\sqrt{3}$
2) 45°-45°-90° triangles pattern: leg lengths equal & hypotenuse is $\sqrt{2}$ times the length of a leg
3) Pair of complementary angles in a rt. triangle, the sine of one angle is the cosine of its
complement.

4) Pair of complementary angles in a rt. triangle, the tangent of one angle is the reciprocal of the tangent of its complement.

For examples & help with vocabulary, visit: <a href="http://intermath.coe.uga.edu/">http://intermath.coe.uga.edu/</a>

### Web Resources

- <u>https://mathbitsnotebook.com/Geometry/RightTriangles/RT306090.html</u> special right triangles
- <u>https://www.cliffsnotes.com/study-guides/geometry/right-triangles/special-right-triangles</u> special right triangles
- http://www.beaconlearningcenter.com/documents/1688\_01.pdf -special right triangles
- <u>http://www.purplemath.com/modules/basirati.htm</u> -trigonometry ratios
- <u>http://www.themathlab.com/toolbox/geometry%20stuff/trigratios.htm</u> -trig. table
- <u>http://hotmath.com/hotmath\_help/topics/trigonometric-ratios.html</u> -trig ratio short notes

#### **Practice**

1. What are the measurements of x, y, q and z?



2. A man is walking his dog on level ground in a straight line with the dog's favorite tree. The angle of elevation from the man's present position to the top of a nearby telephone pole is 30°. The angle of elevation from the tree to the top of the telephone pole is 45°. If the telephone pole is 40 feet tall, how far is the man with the dog from the tree? Express answer to the nearest tenth of a foot.



- 3. Find the exact value of: cos 60° + sin 30° tan 45°.
- 4. Find to the *nearest degree*, the measure of an acute angle formed by the x-axis and the line containing the points (4,3) and (8,9).
- 5. In  $\triangle ABC$ , m<B = 80°, m<C = 34° and a = 16. Find the length of b to the nearest tenth



#### Answers:

- 1. x=12√3; y=12; q=6; z=18
- 2. **29.3 ft**
- з. О
- 4. 56°
- 5. ≈17.2



Accelerated Geometry B/Algebra II Teaching & Learning Framework									
	Block Schedule								
Unit 1 3 weeks	Unit 2 1.5 weeks	Unit 3 1.5 weeks	Unit 4 1.5 weeks	Unit 5 1.5 weeks	<b>Unit 6</b> 2 weeks	Unit 7 2.5 weeks	<b>Unit 8</b> 2 weeks	Unit 9 2.5 weeks	
Circles & Volume	Geometric & Algebraic Connections	Applications of Probability	Quadratics Revisited	Operations with Polynomials	Polynomial Functions	Rational & Radical Relationships	Exponential & Logarithms	Mathematical Modeling	
MGSE9-12.G.C.1-2 (Similar circles; radii, chords, tangents & secants with inscribed, central & circumscribed angles) MGSE9-12.G.C.3-5 (Constructing inscribed & circumscribed circles; construct a tangent line; derive arc lengths) MGSE9-12.G.GMD.1 (Informal arguments for geometric formulas) MGSE9-12.G.GMD.2-4 (Cavalieri's principle; volume; cross-sections & rotations)	MGSE9- 12.G.MG.1-3 (Describe objects; density; design problems) MGSE9- 12.G.GPE.1 (Derive the equation of a circle) MGSE9- 12.G.GPE.4 (Coordinates to prove simple geometric theorems) MGSE9- 12.G.GPE.5-7 (Prove the slope criteria; partition a line segment; compute perimeters using the distance formula)	MGSE9- 12.S.CP.1-4 (Set theory; independent probability; conditional probability; two- way tables) MGSE9-12.S.CP.5 (Recognize & explain conditional probability) MGSE9-12.S.CP.6- 7 (Probability of compound events)	MGSE9-12.N.CN.1 (Complex numbers) MGSE9-12.N.CN.2 (Complex numbers & properties) MGSE9-12.N.CN.3 (Conjugate of complex numbers) MGSE9-12.N.CN.7 (Solve quadratics with complex solutions) MGSE9-12.N.CN.8 (Factoring with complex solutions) MGSE9-12.N.CN.8 (Factoring with complex solutions) MGSE9- 12.A.REI.4 (Solve quadratics in 1 variable) MGSE9- 12.A.REI.4 (Solve quadratic equations by inspection) MGSE9- 12.N.RN.1 (Rational exponents) MGSE9- 12.N.RN.2 (Expressions with radicals & rational exponents)	MGSE9-12.A.APR.1 (Add, subtract & multiply polynomials) MGSE9-12.A.APR.5 (Binomial Theorem) MGSE9-12.A.APR.6 (Rewrite rational expressions) MGSE9-12.F.BF.1 (Write a function) MGSE9-12.F.BF.1b (Combine standard functions) MGSE9-12.F.BF.1c (Compose functions) MGSE9-12.F.BF.4 (Inverse functions) MGSE9-12.F.BF.4a (f(x)=c & inverse) MGSE9-12.F.BF.4b (Use composition to verify inverses) MGSE9-12.F.BF.4c (Values of inverse function from graph or table)	MGSE9-12.N.CN.9 (Fundamental Theorem of Algebra) MGSE9-12.A.SSE.1,a,b (Interpret expressions; Interpret parts & terms of expressions) MGSE9-12.A.SSE.2 (Equivalent expressions) MGSE9-12.A.APR.2 (Remainder Theorem) MGSE9-12.A.APR.3 (Identify zeros) MGSE9-12.F.IF.4 (Polynomial Identities) MGSE9-12.F.IF.4 (Characteristics of functions) MGSE9-12.F.IF.7 (Graph functions) MGSE9-12.F.IF.7 (Graph polynomial functions)	MGSE9-12.A.APR.7 (Rewrite rational expressions) MGSE9-12.A.CED.1 (Create equations & inequalities-1 variable) MGSE9-12.A.CED.2 (create equations & inequalities-2 variables) MGSE9-12.A.REI.2 (Solve simple radical & rational equations) MGSE9-12.F.IF.4 (Characteristics of functions) MGSE9-12.F.IF.5 (Domains of functions) MGSE9-12.F.IF.7 (Graph Functions) MGSE9-12.F.IF.7b (Graph square rt, cube rt, piecewise, step & absolute value functions) MGSE9-12.F.IF.7d (Graph rational functions)	MGSE9-12.A.SSE.3 (Equivalent expressions) MGSE9-12.A.SSE.3c (Properties of exponents) MGSE9-12.F.IF.7 (Graph functions) MGSE9-12.F.IF.7 (Graph exponential & logarithmic functions) MGSE9-12.F.IF.8 (Write a function) MGSE9-12.F.IF.8b (Interpret expressions) MGSE9-12.F.BF.5 (Inverse relationships) MGSE9-12.F.LE.4 (Express exponential models as logarithmic)	MGSE9-12.A.SSE.4 (Derive formula for sum of finite geometric series) MGSE9-12.A.CED.1 (Create equations & inequalities-1 variable) MGSE9-12.A.CED.2 (create equations & inequalities-2 variables) MGSE9-12.A.CED.3 (Represent constraints) MGSE9-12.A.CED.4 (Rearrange formulas) MGSE9-12.A.REI.11 (Solutions to equations) MGSE9-12.F.IF.6 (Average rate of change) MGSE9-12.F.IF.9 (Compare 2 functions) MGSE9-12.F.IF.3 (Build new functions from existing functions) <u>Review:</u> All standards by differentiating for student needs <u>Extend:</u> MGSE9-2.N.CN.4 (Complex plane)	
	These units w	ere written to build u	pon concepts from All units will inclu	prior units, so later u de the Mathematical	inits contain tasks that dep Practices and indicate skill	end upon the concepts addre s to maintain	essed in earlier units.		

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Functions Strand: IF = Interpreting Functions, LE = Linear and Exponential Models, BF = Building Functions, TF = Trigonometric Functions

Geometry Strand: CO = Congruence, SRT = Similarity, Right Triangles, and Trigonometry, C = Circles, GPE = Expressing Geometric Properties with Equations, GMD = Geometric Measurement and Dimension, MG = Modeling with Geometry

Statistics and Probability Strand: ID = Interpreting Categorical and Quantitative Data, IC = Making Inferences and Justifying Conclusions, CP = Conditional Probability and the Rules of Probability, MD = Using Probability to Make Decisions



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## Acc. Algebra I/Geometry A Unit 1: Relationships Between Quantities & Expression

#### Volume 1 Issue 1

### References

HMH Georgia Acc. Coordinate Algebra Text: Unit 1: Modules 1-2

Check with you teacher for online and print access:

Online website: my.hrw.com

### **Web Resources**

- Rational & irrational https://www.illustrative mathematics.org/conten t-standards/tasks/608
- Simplifying radicals http://cms.gavirtualscho ol.org/Shared/Math/GSE Algl16/GSEAlgl\_Relations hipsandExp\_Shared/GSE Algl\_RelationshipsandExp \_Shared8.html#headingt aglink\_1
- Unit conversions

https://www.khanacade my.org/math/prealgebra/rates-andratios/metric-systemtutorial/v/unitconversion

- Polynomials http://mathbitsnotebook
   .com/Algebra1/Polynomi als/POoutline.html
- Polynomials
- http://www.brightstorm. com/search/?k=polynomi als

### **Dear Parents**

Below you will find a list of concepts that your child will use and understand while completing Unit 1: Relationships Between Quantities & Expressions. Also included are references, vocabulary and examples that will help you assist your child at home.

## Concepts Students will Use and Understand

- The structure of expressions and the meaning of their parts in context.
- Appropriateness of units of measure within context.
- Similarities between the system of polynomials and the system of integers.
- Addition, Subtraction, and Multiplication of polynomials is closed.
- Properties of rational and irrational numbers.
- Simplify and/or use the operations of addition, subtraction, and multiplication, with radicals within expressions limited to square roots.
- Visual representation of radicals.

### Vocabulary

- Binomial Expression: An algebraic expression with two unlike terms.
- **Capacity:** The greatest volume that a container can hold.
- **Coefficient:** A number multiplied by a variable.
- **Constant Term:** A quantity that does not change its value.
- **Factor:** When two or more integers are multiplied, each integer is a factor of the product. "To factor" means to write the number or term as a product of its factors.
- Irrational Number: A number whose decimal form is nonterminating and nonrepeating. Irrational numbers cannot be written in the form a/b, where a and b are integers (b cannot be zero). So all numbers that are not rational are irrational.
- Monomial Expression: An algebraic expression with one term.
- Polynomial function: A polynomial function is defined as a function,

 $f(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \dots + a_{n-2} x^2 + a_{n-1} x^1 + a_n$ , where the coefficients are real numbers.

- **Pythagorean Theorem:** It is a theorem that states a relationship that exists in any right triangle. If the lengths of the legs in the right triangle are *a* and *b* and the length of the hypotenuse is *c*, we can write the theorem as the following equation:  $a^2 + b^2 = c^2$
- **Radical:** The symbol,  $\sqrt[b]{a}$ , which is read "the bth root of a," is called a radical.
- **Radicand:** The number underneath the root symbol. So, in  $\sqrt[b]{a}$ , the *a* is called the radicand.
- **Rational Number:** A number expressible in the form *a/b* or *a/b* for some fraction *a/b*. The rational numbers include the integers.
- **Standard Form of a Polynomial**: To express a polynomial by putting the terms in descending exponent order.
- Term: A number, a variable, or a product of numbers and variables.
- **Trinomial**: An algebraic expression with three unlike terms.

## Algebra 1 Unit 1 Practice Problems

Example 1
A rectangle is 5m longer than it is wide. The perimeter is 38m. Find the length & width.
Example 2 Determine if $4 + \sqrt{7} = \frac{a}{b}$ is rational or irrational.
Example 3 What is the simplified form of $\sqrt{98}$ ?

Example 4 Find the difference. Write the answer in standard form.  $(-6x^3 + 5x - 3) - (2x^3 + 4x^2 - 3x + 1)$ 

#### Example 5

A rectangle has a width of (x + 2) and a height of (2x + 1). Find an expression that represents the area as a whole.

## **Answer Key**

**Example 1** 2(w) + 2(w+5)=4w + 10; 4w + 10=38; w=7; the width is 7 and the length is 12

#### Example 2

Irrational

Example 3  $7\sqrt{2}$ 

**Example 4**  $-8x^3 - 4x^2 + 8x - 4$ 

**Example 5**  $2x^2 + 5x + 2$ 



## Accelerated Geometry B/Algebra II: Unit 2 Geometric & Algebraic Connections

#### References

#### Textbook:

- HMH Acc. Analytic Geometry B/Adv. Alg., Unit 6
- HMH Coordinate Algebra, Unit 6
- HMH Advanced Algebra, Unit 6

#### **Online Access:**

http://www.my.hrw.com

#### **Helpful Links:**

- Circle Equations: http://www.purplem ath.com/modules/sq rcircle.htm
- Area and Perimeter on a Grid:

https://mathbits notebook.com/G eometry/Coordin ateGeometry/CG Area.html

- Distance Formula: https://mathbitsnote book.com/Geometry /CoordinateGeometr y/CGdistance.html
- Partition a Line Segment: https://mathbitsnote book.com/Geometry /CoordinateGeometr y/CGdirectedsegmen ts.html
- Coordinate Geometry Proofs: https://mathbitsnote book.com/Geometry /CoordinateGeometr

Dear Parents,

Students will use the concepts of distance, midpoint, and slope to verify algebraically geometric relationships of figures in the coordinate plane (triangles, quadrilaterals, and circles). Students will solve problems involving parallel and perpendicular lines, perimeters and areas of polygons, and the partitioning of a segment in a given ratio. Students will derive the equation of a circle and model real-world objects using geometric shapes and concepts.

#### **Concepts Students will Use & Understand**

- prove the slope relationship that exists between parallel lines and between perpendicular lines and then use those relationships to write the equations of lines
- extend the Pythagorean Theorem to the coordinate plane
- develop and use the formulas for the distance between two points and for finding the point that partitions a line segment in a given ratio
- revisit definitions of polygons while using slope and distance on the coordinate plane
- use coordinate algebra to determine perimeter and area of defined figures
- use Algebra to model Geometric ideas
- spend time developing equations from geometric definition of circles
- address equations in standard and general forms
- graph by hand and by using graphing technology
- develop the idea of algebraic proof in conjunction with writing formal geometric proofs

#### Vocabulary

0

- Distance Formula:  $d = \sqrt{(x_2 x_1)^2 + (y_2 y_1)^2}$
- Formula for finding the point that partitions a directed segment AB at the ratio of a : b from A(x<sub>1</sub>, y<sub>1</sub>) to B(x<sub>2</sub>, y<sub>2</sub>):

$$x_1 + \frac{a}{a+b}(x_2 - x_1), y_1 + \frac{a}{a+b}(y_2 - y_1)$$

r 
$$\left(\frac{a}{a+b}(x_2-x_1)+x_1, \frac{a}{a+b}(y_2-y_1)+y_1\right)$$

or 
$$\left(\frac{bx_1 + ax_2}{b + a}, \frac{by_1 + ay_2}{b + a}\right) \leftarrow$$
 weighted average approach

- **Center of a Circle**: The point inside the circle that is the same distance from all of the points on the circle.
- **Circle:** The set of all points in a plane that are the same distance, called the radius, from a given point, called the center. Standard form:  $(x h)^2 + (y k)^2 = r^2$

y/CGShowProofs.htm

- **Diameter**: The distance across a circle through its center. The line segment that includes the center and whose endpoints lie on the circle.
- **Pythagorean Theorem**: A theorem that states that in a right triangle, the square of the length of the hypotenuse equals the sum of the squares of the lengths of the legs.
- **Radius:** The distance from the center of a circle to any point on the circle. Also, the line segment that has the center of the circle as one endpoint and a point on the circle as the other endpoint.
- Standard Form of a Circle:  $(x h)^2 + (y k)^2 = r^2$ , where (h,k) is the center and r is the radius.

### **Sample Practice Problems**

#### Example 1:

Write the standard form of the equation of a circle that passes through the given point (7,-4) and whose center is at the origin.

#### Example 2:

Show that A(2, -1), B(1, 3), C(6, 5), and D(7, 1) are the vertices of a parallelogram.



#### Key :

**Example 1**:  $x^2 + y^2 = 65$ **Example 2**:

#### SOLUTION

There are many ways to solve this problem.

**Method 1** Show that opposite sides have the same slope, so they are parallel.

Slope of  $\overline{AB} = \frac{3 - (-1)}{1 - 2} = -4$ Slope of  $\overline{CD} = \frac{1 - 5}{7 - 6} = -4$ Slope of  $\overline{BC} = \frac{5 - 3}{6 - 1} = \frac{2}{5}$ Slope of  $\overline{DA} = \frac{-1 - 1}{2 - 7} = \frac{2}{5}$ 

 $\overline{AB}$  and  $\overline{CD}$  have the same slope so they are parallel. Similarly,  $\overline{BC} \parallel \overline{DA}$ .

Method 2 Show that opposite sides have the same length.

$$AB = \sqrt{(1-2)^2 + [3-(-1)]^2} = \sqrt{17}$$
$$CD = \sqrt{(7-6)^2 + (1-5)^2} = \sqrt{17}$$
$$BC = \sqrt{(6-1)^2 + (5-3)^2} = \sqrt{29}$$
$$DA = \sqrt{(2-7)^2 + (-1-1)^2} = \sqrt{29}$$

AB  $\cong \overline{CD}$  and  $\overline{BC} \cong \overline{DA}$ . Because both pairs of opposite sides are congruent, *ABCD* is a parallelogram.

Method 3 Show that one pair of opposite sides is congruent and parallel.

Find the slopes and lengths of  $\overline{AB}$  and  $\overline{CD}$  as shown in Methods 1 and 2.

Slope of 
$$\overline{AB}$$
 = Slope of  $\overline{CD}$  = -4

$$AB = CD = \sqrt{17}$$

- $\overrightarrow{AB}$  and  $\overrightarrow{CD}$  are congruent and parallel, so *ABCD* is a parallelogram.
- Because opposite sides are parallel, ABCD is a parallelogram.



## Accelerated Geometry B/Algebra II Unit 3 Applications of Probability

### References

#### Textbook Connection: HMH Acc. Analytic Geometry B/Adv. Alg. Unit 7 Online Access: http://www.my.hrw.com

### Dear Parents

In this unit, students will take their previously acquired knowledge of probability for simple and compound events and expand that to include conditional probabilities (events that depend upon and interact with other events) and independence. Students will be exposed to elementary set theory and notation (sets, subsets, intersection and unions). Finally, students will use their knowledge of conditional probability and independence to make determinations on whether or not certain variables are independent.

### **Concepts Students will Use & Understand**

- Use set notation as a way to algebraically represent complex networks of events or real world objects.
- Represent everyday occurrences mathematically through the use of unions, intersections, complements and their sets and subsets.
- Use Venn Diagrams to represent the interactions between different sets, events or probabilities.
- Find conditional probabilities by using a formula or a two-way frequency table.
- Understand independence as conditional probabilities where the conditions are irrelevant.
- Analyze games of chance, business decisions, public health issues and a variety of other parts of everyday life can be with probability.
- Model situations involving conditional probability with two-way frequency tables and/or Venn Diagrams.
- Confirm independence of variables by comparing the product of their probabilities with the probability of their intersection.

## Vocabulary

- **Complement:** Given a set A, the complement of A, denoted  $\overline{A}$  or A', is the set of elements that are not members of A.
- **Conditional Probability:** The probability of an event A, given that another event, B, has already occurred; denoted P(A | B).
- **Dependent Events:** Two or more events in which the outcome of one event affects the outcome of the other event or events.
- Element: A member or item in a set.
- Independent Events: Events whose outcomes do not influence each other.
- Intersection of Sets: The set of all elements contained in all of the given sets, denoted  $\cap$ .
- **Outcome:** A possible result of an experiment.
- Sample Space: The set of all possible outcomes from an experiment.
- Set: A collection of numbers, geometric figures, letters, or other objects that have some characteristic in common.
- Subset: a set in which every element is also contained in a larger set.
- Union of Sets: The set of all elements that belong to at least one of the given two or more sets denoted ∪.
- Venn Diagram: A picture that illustrates the relationship between two or more sets.

#### Helpful Links:

#### □ Set Notation:

- Venn Diagrams and Set Notation:
- Conditional Probability:
- Two way Frequency Tables:
- Conditional Probability and Independence:

#### **Sample Practice Problems:**



2. Suppose a study of speeding violations and drivers who use car phones produced the following fictional data:

	Speeding violation in the last year	No speeding violation in the last year	Total
Car phone user	25	280	305
Not a car phone user	45	405	450
Total	70	685	755

- P(person is a car phone user) a.
- b. P(person had no violation in the last year)
- P( person had no violation in the last year AND was a car phone user) c.
- d. P(person is a car phone user GIVEN that they had a violation in the past year)
- If there is a 10% chance that the moon will be in the Seventh House and Jupiter will also align with Mars, and a 25% chance that Jupiter 3. will align with Mars, then what is the probability that the Moon is in the Seventh House given that Jupiter aligns with Mars?

#### Solutions:

**a**  $A = \{b, d, e, h\}$  **b**  $B = \{e, f, h, i, j\}$ **c**  $A' = \{a, c, f, g, i, j, k\}$  **d**  $B' = \{a, b, c, d, g, k\}$ 1. •  $A \cap B = \{e, h\}$  f  $A \cup B = \{b, d, e, f, h, i, j\}$ **g**  $(A \cup B)' = \{a, c, g, k\}$ **h**  $A' \cup B' = \{a, b, c, d, f, g, i, j, k\}$ 

 $\frac{\text{number of car phone users}}{\text{total number in study}} = \frac{305}{755}$ 2.

b. 
$$\frac{\text{number that had no violation}}{\text{total number in study}} = \frac{685}{755}$$

280755 c.

> 25 70 (The sample space is reduced to the number of persons who had a violation.)

3. Let: M=The Moon is in the Seventh House and J= Jupiter aligns with Mars, then

 $M|J) = P(M \cap J)/P(J) = .10/.25 = 0.4$ 



## Accelerated Geometry B/Algebra II Unit 4: Quadratics Revisited

## References

**Textbook Connection:** 

**HMH Georgia Analytic** 

**Geometry B/Advanced** 

Algebra Text:

Units 1 & 2

### Dear Parents,

In this unit students will:

- Define rational exponents
- Rewrite expression involving radicals and rational exponents
- Define the imaginary number i
- Define complex numbers
- Operate with complex numbers
- Understand that the basic properties of numbers continue to hold with expressions involving exponents.

### **Concepts Students will Use & Understand**

- Extend the properties of exponents to rational exponents.
- Rewrite expressions involving radicals & rational exponents.
- Use properties of rational & irrational numbers to find the sum and product.
- Perform arithmetic operations with complex numbers.
- Find the conjugate of a complex number to find quotients of complex numbers

### Vocabulary

• **Complex number**: A complex number is the sum of a real number and an imaginary number (a number whose square is a real number less than zero), i.e. an

expression of the form a + bi, where *a* and *b* are real numbers and *i* is the *imaginary unit*, satisfying  $i^2 = -1$ .

- **Exponential functions:** A function of the form  $y = a \cdot b^x$  where a > 0 and either 0 < b < 1 or b > 1.
- <u>Nth roots</u>: The number that must be multiplied by itself *n* times to equal a given value. The *n*th root can be notated with radicals and indices or with rational exponents, i.e.  $x^{1/3}$  means the cube root of *x*.
- **Polynomial function** A **polynomial function** is defined as a function,  $f(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots + a_{n-2} x^2 + a_{n-3} x^1 + a_n$ , where the coefficients are real numbers.
- **<u>Rational exponents:</u>** For *a* > 0, and integers *m* and *n*, with *n* > 0,

$$a^{\frac{m}{n}} = \sqrt[m]{a^m} = (\sqrt[n]{a})^m; a^{m/n} = (a^{1/n})^m = (a^m)^{1/n}$$

- **<u>Rational expression</u>**: A quotient of two polynomials with a non-zero denominator.
- **<u>Rational number</u>**: A number expressible in the form a/b or -a/b for some fraction a/b. The rational numbers include the integers.
- For further help: http://intermath.coe.uga.edu/dictnary/homepg.asp http://www.amathsdictionaryforkids.com/

# online textbook resource:

Every student will receive a

text copy and access to the

#### http://my.hrw.com/

#### Helpful Links:

### • Lesson on Complex

Numbers:

http://www.purplemath.co m/modules/complex.htm

## • Lesson on Computing Polynomials:

http://www.purplemath.co m/modules/polymult.htm

## • Lesson on Rational Exponents:

http://www.themathpage.c om/Alg/rationalexponents.htm

#### Lesson on Operations on Complex Numbers:

https://www.khanacademy .org/math/algebra2/introdu ction-to-complex-numbersalgebra-2/the-complexnumbers-algebra-2/v/complex-number-intro

### **Sample Practice Problems**

#### Example 1

Combine like terms: (2+3i)+(7+i)

Answer: 9+4i



Find the conjugate & calculate the quotient:  $\frac{(2+5i)}{(5+2i)}$ 



Example 3

Find the y-coordinate for the following:  $y = 2(x - 4)^2 - 5$  for x = 2.

Answer: y = 3

Example 4

Give the value of the discriminant of the equation  $4x^2 - 8x = -4$ A) 0 B) 1 C) -128 D) 128



Graph. Identify zeros, intervals of increase, intervals of decrease, vertex and axis of symmetry.







## Accelerated Geometry B/Algebra II Unit 5: Operations with Polynomials

#### References

#### Textbook:

 HMH Geometry B/Advanced Algebra, Unit 6: Modules 10 & 11

#### Online Access:

http://www.my.hrw.com

#### **Helpful Links:**

•

Khan Academy: https://www.khanac ademy.org/math/alg ebra2/polynomial\_an d\_rational/polynomi al\_tutorial/v/termscoefficients-andexponents-in-apolynomial

#### • Binomial Theorem:

https://www.khanac ademy.org/math/alg ebra2/polynomial\_an d\_rational/binomial\_ theorem/v/binomialtheorem

#### • Dividing Polynomials:

https://www.khanac ademy.org/math/alg ebra2/polynomial\_an d\_rational/dividing\_p olynomials/v/polyno mial-division

#### Synthetic Division:

https://www.khanac ademy.org/math/alg ebra2/polynomial\_an d\_rational/syntheticdivision/v/syntheticdivision

#### Dear Parents,

This unit develops the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers, and division of polynomials with long division of integers. Students will find inverse functions and verify by composition that one function is the inverse of another function.

#### **Concepts Students will Use & Understand**

- understand the definition of a polynomial
- interpret the structure and parts of a polynomial expression including terms, factors, and coefficients
- simplify polynomial expressions by performing operations, applying the distributive property, and combining like terms
- use the structure of polynomials to identify ways to rewrite them and write polynomials in equivalent forms to solve problems
- perform arithmetic operations on polynomials and understand how closure applies under addition, subtraction, and multiplication
- divide one polynomial by another using long division
- use Pascal's Triangle to determine coefficients of binomial expansion
- use polynomial identities to solve problems
- use complex numbers in polynomial identities and equations
- find inverses of simple functions

#### Vocabulary

- **Coefficient:** a number multiplied by a variable.
- **Degree:** the greatest exponent of its variable
- End Behavior: the value of f(x) as x approaches positive and negative infinity
- **Pascal's Triangle:** an arrangement of the values of  ${}_{n}C_{r}$  in a triangular pattern where each row corresponds to a value of n
- **Polynomial:** a mathematical expression involving a sum of nonnegative integer powers in one or more variables multiplied by coefficients. A polynomial in one variable with constant coefficients can be written in  $p_{n}^{n} + p_{n} + p_{n}^{n-1} + p_{n} + p_{n}^{2} + p_{n} + p_{n} + p_{n}^{2}$

### $a_n x^n + a_{n-1} x^{n-1} + \dots a_2 x^2 + a_1 x + a_0$ form.

- **Remainder Theorem:** states that the remainder of a polynomial f(x) divided by a linear divisor (x c) is equal to f(c).
- Synthetic Division: Synthetic division is a shortcut method for dividing a
  polynomial by a linear factor of the form (x a). It can be used in place of the
  standard long division algorithm.
- **Roots**: solutions to polynomial equations.
- **Zero**: If f(x) is a polynomial function, then the values of x for which f(x) = 0 are called the **zeros** of the function. Graphically, these are the x intercepts.

### Sample Problems

Find the following products. Be sure to simplify results.

a. $3x(2x^2+8x+9)$	b. $-2x^2(5x^2-x-4)$
$6x^3 + 24x^2 + 27x$	$-10x^4 + 2x^3 + 8x^2$
c. $(2x+7)(2x-5)$	d. $(4x-7)(3x-2)$
$4x^2+4x-35$	$12x^2 - 29x + 14$
e. $(x-3)(2x^2+3x-1)$	f. $(6x+4)(x^2-3x+2)$
$2x^3 - 3x^2 - 10x + 3$	$6x^3 - 14x^2 + 8$
g. $(4x-7y)(4x+7y)$	h. $(3x-4)^2$
$16x^2 - 49y^2$	$9x^2 - 24x + 16$
i. $(x-1)^3$	j. $(x-1)^4$
$x^3 - 3x^2 + 3x - 1$	$x^4 - 4x^3 + 6x^2 - 4x - 1$

Description	Identity
Difference of Two Squares	$(a+b)(a-b) = \mathbf{a}^2 - \mathbf{b}^2$
Sum of Two Squares	$(a+bi)(a-bi) = \mathbf{a^2} + \mathbf{b^2}$
Perfect Square Trinomial	$(a+b)^2 = a^2 + 2ab + b^2$
Perfect Square Trinomial	$(a-b)^2 = a^2 - 2ab + b^2$
Binomial Cubed	$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
Binomial Cubed	$(a-b)^3 = a^3 - 3a^2b + 3ab^2 + b^3$
Sum of Two Cubes	$a^{3} + b^{3} = (a + b) (a^{2} - ab + b^{2})$
Difference of Two Cubes	$a^{3}-b^{3} = (a - b) (a^{2} + ab + b^{2})$

Divide:  $x - 2 \sqrt{x^3 + 2x^2 - 5x - 6}$ 

Long Division vs. Synthetic Division

$$\begin{array}{r} 4x^3 + 5x^2 + 3x + 2 \\ \underline{x-2} 4x^4 - 3x^3 - 7x^2 - 4x - 9 \\ \underline{4x^4 - 8x^3} \\ 5x^3 - 7x^2 \\ \underline{5x^3 - 10x^2} \\ 3x^2 - 4x \\ \underline{3x^2 - 6x} \\ 2x - 9 \\ \underline{2x - 4} \\ -5 \end{array}$$

$$\begin{array}{r} 4 & 5 & 3 & 2 \\ \underline{-2} 4 - 3 - 7 - 4 - 9 \\ \underline{-3} \\ 5 \\ 5 \\ 5 \\ \underline{-10} \\ 3 \\ \underline{-10} \\ 3 \\ \underline{-10} \\ 3 \\ \underline{-4} \\ -5 \end{array}$$



## **Accelerated Geometry B/Algebra II Unit 6: Polynomials Functions**

#### References

**HMH** Geometry

Unit 6 Module 11

B/Advanced Algebra,

Textbook:

**Online Access:** 

**Helpful Links:** 

http://www.my.hrw.com

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#### Dear Parents,

In this unit, students continue their study of polynomials by identifying zeros and making connections between zeros of a polynomial and solutions of a polynomial equation. Students will see how the Fundamental Theorem of Algebra can be used to determine the number of solutions of a polynomial equation and will find all the roots of those equations. Students will graph polynomial functions and interpret the key characteristics of the function

#### **Concepts Students will Use & Understand**

- use polynomial identities to solve problems
- use complex numbers in polynomial identities and equations •
- understand and apply the rational Root Theorem •
- understand and apply the Remainder Theorem •
- understand and apply The Fundamental Theorem of Algebra
- understand the relationship between zeros and factors of polynomials
- represent, analyze, and solve polynomial functions algebraically and graphically •

#### Vocabulary

**End Behavior:** the value of f(x) as x approaches positive and negative infinity Relative Minimum: a point on the graph where the function is increasing as you move away from the point in the positive and negative direction along the horizontal axis. Relative Maximum: a point on the graph where the function is decreasing as you move away from the point in the positive and negative direction along the horizontal axis. Fundamental Theorem of Algebra: every non-zero single-variable polynomial with complex coefficients has exactly as many complex roots as its degree, if each root is counted up to its multiplicity.

**Multiplicity:** the number of times a root occurs at a given point of a polynomial equation.

**Pascal's Triangle:** an arrangement of the values of  ${}_{n}C_{r}$  in a triangular pattern where each row corresponds to a value of n

Rational Root Theorem: a theorem that provides a complete list of all possible rational roots of a polynomial equation. It states that every rational zero of the polynomial equation  $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$ , where all coefficients are integers, has the following form:  $\frac{p}{a} = \frac{\text{factors of constant term } a_0}{a}$ 

q factors of leading coefficient  $a_n$ 

**Remainder Theorem:** states that the remainder of a polynomial f(x) divided by a linear divisor (x - c) is equal to f(c)

- **GA Virtual Learning** • http://cms.gavirtualsch ool.org/Shared/Math/G SEAdvancedAlgebra/P olynomialFunctions/in dex.html
- Fundamental Theorem of Algebra ook.com/Algebra2/Qua dratics/QDfundamenta
- The Polynomial Remainder Theorem
- **Polynomial Identities** • ynomials/POIdentity.ht
- Graph Polynomial • Functions

ook.com/Algebra2/Pol

**MathBitsNotebook** Algebra 2 http://mathbitsnotebo ok.com/Algebra2/Alge

#### **Sample Problems**

1. The height of an arrow shot by a 6 foot tall person is given by the function equation image indicator where h is the height and t is the time. At what time would the arrow be able to hit a target 10 feet in the air?

The arrow could hit a 10 foot target in 2 sec. or in 2 2/3 sec.

2. Draw a rough sketch of the graph of  $y = -x^2 + 4x - 3$ 



3. A soccer ball is kicked from the ground. The height of the ball is modeled by the equation  $h(t) = -4.9t^2 + 19.6t$ Height is in meters. Time is in seconds. How long is the ball in the air?

#### <mark>4 seconds</mark>

4. Describe the key features of the following polynomial function:  $f(x) = x^4 + x^2 - 20$ 



(0, -20) End behavior:

 $x \to -\infty, f(x) \to \infty; x \to \infty, f(x) \to \infty$ 



## Accelerated Geometry B/Algebra II Unit 7: Radical & Rational Relationships

#### References

**HMH** Geometry

B/Advanced Algebra,

Unit 7 Modules 13-14

**GA Virtual Learning** 

http://cms.gavirtuals

chool.org/Shared/Ma

th/GSEAdvancedAlge

bra/RationalAndRadi

calRelationships/inde

**Textbook:** 

**Online Access:** 

**Helpful Links:** 

•

•

http://www.my.hrw.com

x.html

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#### **Dear Parents,**

Students investigate two more types of families of functions – radical and rational functions. They also learn how to simplify expressions and solve equations and inequalities that involve rational and radical expressions. Below you find will several resources that can be used to help support the learning for your child.

#### **Concepts Students will Use & Understand**

- Explore Rational and Radical Functions
- Determine rational numbers extend the arithmetic of integers by allowing division by all numbers except zero. Similarly, rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial
- Notice the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers
- Investigate the properties of simple rational and radical functions and then expand their knowledge of the graphical behavior and characteristics of more complex rational functions
- Recall and make use of their knowledge of polynomial functions as well as compositions of functions to investigate the characteristics of these more complex rational functions
- Solve equations and inequalities involving rational and radical functions
- Understand that not all solutions generated algebraically are actually solutions to the equations and extraneous solutions will be explored
- Apply these rational and radical functions with an emphasis on interpretation of real world phenomena as it relates to certain characteristics of the rational expressions

#### Vocabulary

- **Radical Function:** A function containing a root. The most common radical functions are the square root and cube root functions:  $f(x) = \sqrt{x}$  and  $g(x) = \sqrt[3]{x}$ .
- **Rational Function:** The quotient of two polynomials, P(z) and Q(z), where  $R(z) = \frac{P(z)}{Q(z)}$

- Khan Academy https://www.khanac ademy.org/math/alg ebra2/rationalexpressionsequations-andfunctions
- Radical Functions http://www.purplem ath.com/modules/gr aphrad.htm
- Rational Functions:

http://www.purplem ath.com/modules/gr phrtnl.htm

#### Sample Problems

Simplify each expression or solve each equation or inequality.



4. Graph:  $f(x) = \sqrt[3]{x} + 1$ . State Domain and Range.



- 5. f(x) is a radical function whose domain is (- $\infty$ , 5] and whose range is (- $\infty$ , 0], and  $g(x) = \sqrt{x+5}+2$ . Which statement is true for both f(x) and g(x)?
  - A. g(x) has a higher y-intercept than f(x)
  - B. g(x) has a greater zero than f(x)
  - C. The functions have the same domain.
  - D. The functions have the same range.

### Answer Key:





## Accelerated Geometry B/Algebra II Unit 8: Exponential & Logarithmic Functions

### References

Textbook Connection: HMH Georgia Analytic Geometry B/Advanced Algebra Text: Unit 8: Modules 15 & 16

Every student will receive a text copy and access to the online textbook resource:

#### http://my.hrw.com/

#### Helpful Links:

#### □ GA Virtual:

http://cms.gavirtualsc hool.org/Shared/Math /GSEAdvancedAlgebra /ExponentialAndLogar ithms/index.html

#### Logarithmic Functions: http://mathbitsnoteb

ook.com/Algebra2/Ex ponential/EXLogFuncti ons.html

#### □ Khan Academy:

https://www.khanaca demy.org/math/algeb ra2/exponential\_and\_ logarithmic\_func

#### Purple Math:

http://www.purplema th.com/modules/grap hlog.htm

#### □ The Math Page:

http://www.themathp age.com/aprecalc/log arithmic-exponentialfunctions.htm

## Dear Parents,

Students extend their work with exponential functions to include solving exponential equations with logarithms. They analyze the relationship between these two functions. In this unit students will:

- Review exponential functions and their graphs
- Explore exponential growth
- Develop the concept of a logarithm as an exponent along with the inverse relationship with exponents
- Define logarithms and natural logarithms
- Develop the change of base formula
- Develop the concept of logarithmic function
- Solving problems relating to exponential functions and logarithms

### **Concepts Students will Use & Understand**

- The concept of a function
- Various representations of functions
- Exponential functions and characteristics of their graphs
- The solution of linear equations using algebra and graphing approaches
- Familiarity with graphing technology
- Use patterns to write a function to model a situation

### Vocabulary

- Asymptote: An asymptote is a line or curve that approaches a given curve arbitrarily closely. A graph never crosses a vertical asymptote, but it may cross a horizontal or oblique asymptote.
- **Common logarithm:** A logarithm with a base of 10. A common logarithm is the exponent, *a*, such that  $10^a = b$ . The common logarithm of *x* is written *log x*. For example, *log 100* = 2 because  $10^2 = 100$ .
- **Continuously compounded interest:** Interest that is, theoretically, computed and added to the balance of an account each instant. The formula is *A* = *Pe*<sup>*rt*</sup>, where *A* is the ending amount, *P* is the principal or initial amount, *r* is the annual interest rate, and *t* is the time in years.
- **Compounded interest:** A method of computing the interest, after a specified time, and adding the interest to the balance of the account. Interest can be computed as little as

once a year to as many times as one would like. The formula is  $A = P(1 + \frac{r}{n})^{nt}$  where A is the ending amount, P is the principal or initial amount, r is the annual interest rate, n is the number of times compounded per year, and t is the number of years.

- **Exponential functions:** A function of the form  $y = a^x$  where a > 0 and  $a \neq 1$ .
- Logarithmic functions: A function of the form  $y = \log_b x$  with  $b \neq 1$  and b and x both positive. A logarithmic function is the inverse of an exponential function. The inverse of  $y = b^x$  is  $y = \log_b x$ .

- **Logarithm:** The logarithm base *b* of a number *x*,  $\log_b x$ , is the exponent to which *b* must be raised to equal *x*.
- Natural exponential: Exponential expressions or functions with a base of e; i.e.,  $y = e^x$ .
- Natural logarithm: A logarithm with a base of *e*. *ln b* is the exponent, *a*, such that *e<sup>a</sup>* = *b*. The natural logarithm of *x* is written *ln x* and represents log<sub>*e*</sub> *x*. For example, *ln 8* = 2.0794415... because *e*<sup>2.0794415...</sup> = 8.

For further help:

http://intermath.coe.uga.edu/dictnary/homepg.asp http://www.amathsdictionaryforkids.com/

### Sample Practice Problems

1) State the domain and range for  $f(x) = -2^x + 4$ 

D: all real numbers; R: y < 4

2) State the domain and range for  $3\log_5 x$ 

D: x > 0; R: all real numbers

3) Solve  $2(3)^{2x} = 5$ x = 0.417

4) Solve  $5\log(x-2) = 11$ x = 160.49

5) Identify asymptotes, y-intercept and point of maximum growth:  $y = \frac{2}{1 + e^{-2x}}$ 

Asymptotes: x-axis and y=2; y-intercept (0,1); maximum growth is at (0,1)

6) Find the inverse of the function  $y = \log_6 x$ 

y = 6x

7) The value of a new car purchased for \$24,900 decreases by 10% per year. Write an exponential decay model for the value of the car. After about how many years will the car be worth half its purchase price?

V(t) = 24,9000(0.90)t; about 6.58 years

8) You deposit \$4,000 in an account that pays 7% annual interest compounded continuously. Find the balance at the end of five years.

<mark>\$5,676.27</mark>



## Accelerated Geometry B/Algebra II Unit 9: Mathematical Modeling

## References

### Dear Parents,

In this unit students will:

- Synthesize and generalize what they have learned about a variety of function families
- derive the formula for the sum of a finite geometric series and use it to solve problems
- Explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect regardless of the type of the underlying functions
- Identify appropriate types of functions to model a situation,
- Adjust parameters to improve the model,
- Compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit
- Determine whether it is best to model with multiple functions creating a piecewise function

### **Concepts Students will Use & Understand**

- quantitative reasoning
- solving various functions (finding zeros) through factoring, using other algebraic processes, using geometry, or by graphing
- properties of exponents and the associated properties of logarithms
- a working knowledge of geometric vocabulary
- writing explicit and recursive formulas for geometric sequences
  - the ability to recall and apply basic algebraic and geometric processes
- an ability to understand mathematics through a variety of representations
- familiarity with technology, particularly the graphing calculator
- prior knowledge and understanding of functions learned earlier in the course, as this is the culminating unit

### Vocabulary

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- Geometric Sequence: is a sequence with a constant ratio between successive terms
- Geometric Series: the expression formed by adding the terms of a geometric sequence
- **Recursive**: A type of sequence in which successive terms are generated by preceding terms in the sequence.
- Sum of a finite geometric series: The sum, S<sub>n</sub>, of the first *n* terms of a geometric sequence is given by  $S_n = \frac{a_1 a_1 r^n}{1 r} = \frac{a_1(1 r^n)}{1 r}$ , where  $a_1$  is the first term and *r* is the common ratio ( $r \neq 1$ ).
- Sum of an infinite geometric series: The general formula for the sum *S* of an infinite

geometric series  $a_1 + a_2 + a_3 + \dots$  with common ratio *r* where |r| < 1 is  $S = \frac{a_1}{1 - r}$ . If an

Textbook Connection: HMH Analytic Geometry B/Advanced Algebra Text: Unit 6: Module 12 & Unit 10: Modules 19-22

Every student will receive a text copy and access to the online textbook resource:

http://my.hrw.com/

#### **Helpful Links:**

#### □ GA Virtual:

http://cms.gavirtualsc hool.org/Shared/Math /GSEAdvancedAlgebra /MathematicalModeli ng/index.html

#### □ Khan Academy:

https://www.khanaca demy.org/math/algeb ra2/functions-andgraphs/piecewisefunctionstutorial/v/graphs-ofabsolute-valuefunctions

#### □ Math Bits Notebook:

https://mathbitsnoteb ook.com/Algebra2/Se quences/SSGeometric. html

#### Math Bits Notebook: http://mathbitsnoteb

ook.com/Algebra1/Fu nctionGraphs/FNGTyp ePiecewise.html

#### □ Wolfram Math World:

http://mathworld.wol fram.com/GeometricS eries.html

#### □ Purple Math:

http://www.purplema th.com/modules/serie s5.htm infinite geometric series has a sum, i.e. if |r| < 1, then the series is called a **convergent** geometric series. All other geometric (and arithmetic) series are **divergent**.

#### For further help:

http://intermath.coe.uga.edu/dictnary/homepg.asp http://www.amathsdictionaryforkids.com/

### Sample Practice Problems

1. The price of dance lesson depends upon the number of lessons that you select. If *x* is the number of lessons then the fee for the lessons (in dollars) can be found using the piecewise function

 $f(x) = \begin{cases} 40x \ if \ 0 < x \le 4\\ 30x \ if \ 4 < x \le 8\\ 25x \ if \ x > 8 \end{cases}$ 

The lessons are increasing by 10% per lesson with a \$5 processing fee for each student. What is the new function for the cost of lessons?

 $f(x) = \begin{cases} 44x + 5 & \text{if } 0 < x \le 4\\ 33x + 5 & \text{if } 4 < x \le 8\\ 27.5x + 5 & \text{if } x > 8 \end{cases}$ 

2. Use the function  $f(x)\sqrt[3]{5x}$  to answer the following questions: What is the domain & range? Domain: all real numbers; Range: all real numbers

What is the inverse of f(x)?  $f^{-1}(x) = \frac{x^3}{5}$ 

What is the domain and range of the inverse function? Domain: all real numbers; Range: all real numbers

Over what line does the function and its inverse reflect across on the coordinate plane? y = x

Identify the axis of symmetry, vertex, intercepts, domain, range, slope, and max/min of the following absolute value function:
 f(x) = |x| + 3

A.O.S: x = 0; Vertex: (0, 3); x-intercept: none; y-intercept: 3; Domain: all real numbers; Range:  $y \ge 3$ ; Left Slope: -1; Right Slope: 1; Minimum: 3



Advanced Mathematical Decision Making (AMDM) Teaching & Learning Framework									
	Sem	ester 1		Semester 2					
Unit 1 4 weeks Analyzing Numerical Data	Unit 2 <sup>5 weeks</sup> Probability	Unit 3 7 weeks Statistical Studies	Unit 4 <sup>5 weeks</sup> Using Recursion in Models & Decision Making	Unit 5 4 weeks Using Functions in Models & Decision Making	Unit 6 <sup>6 weeks</sup> Decision Making in Finance	Unit 7 5 weeks Networks & Graphs			
MAMDMN1. Students will extend the understanding of proportional reasoning, ratios, rates, and percents by applying them to various settings to include business, media, and consumerism. a. Use proportional reasoning to solve problems involving ratios. b. Understand and use averages, weighted averages, and indices. c. Solve problems involving large quantities that are not easily measured. d. Understand how identification numbers, such as UPCs, are created and verified.	MAMDMD1. Students will determine probability and expected value to inform everyday decision making. a. Determine conditional probabilities and probabilities of compound events to make decisions in problem situations. b. Use probabilities to make and justify decisions about risks in everyday life. c. Calculate expected value to analyze mathematical fairness, payoff, and risk.	MAMDMD2. Students will build the skills and vocabulary necessary to analyze and critique reported statistical information, summaries, and graphical displays. MAMDMD3. Students will apply statistical methods to design, conduct, and analyze statistical studies.	MAMDMD4. Students will use functions to model problem situations in both discrete and continuous relationships. a. Determine whether a problem situation involving two quantities is best modeled by a discrete (pattern identification, population growth, compound interest) or continuous (medication dosage, climate change, bone decay) relationship. b. Use linear, exponential, logistic, piecewise and sine functions to construct a model. MAMDMG1. Students will create and use two- and three-dimensional representations of authentic situations. MAMDMG2. Students will solve geometric problems involving inaccessible distances using basic trigonometric principles, including the Law of Cosines.	MAMDMD4. Students will use functions to model problem situations in both discrete and continuous relationships. a. Determine whether a problem situation involving two quantities is best modeled by a discrete (pattern identification, population growth, compound interest) or continuous (medication dosage, climate change, bone decay) relationship. b. Use linear, exponential, logistic, piecewise and sine functions to construct a model. MAMDMG1. Students will create and use two- and three-dimensional representations. MAMDMG2. Students will solve geometric problems involving inaccessible distances using basic trigonometric principles, including the Law of Sines and the Law of Cosines.	MAMDMA3. Students will create and analyze mathematical models to make decisions related to earning, investing, spending, and borrowing money. a. Use exponential functions to model change in a variety of financial situations. b. Determine, represent, and analyze mathematical models for income, expenditures, and various types of loans and investments.	MAMDMA2. Students will use a variety of network models to organize data in quantitative situations, make informed decisions, and solve problems. a. Solve problems represented by a vertex- edge graph, and find critical paths, Euler paths, and minimal spanning trees. b. Construct, analyze, and interpret flow charts to develop an algorithm to describe processes such as quality control procedures. c. Investigate the scheduling of projects using PERT. d. Consider problems that can be resolved by coloring graphs.			

All units will include the Mathematical Practices and indicate skills to maintain.

NOTE: Mathematical standards are interwoven and should be addressed throughout the year in as many different units and tasks as possible in order to stress the natural connections that exist among mathematical topics. Revised standards are in red font.



Advanced Mathematical Decision Making (AMDM) Teaching & Learning Framework								
Block Schedule								
Unit 1 2 weeks Analyzing Numerical Data	Unit 2 2.5 weeks Probability	Unit 3 3.5 weeks Statistical Studies	Unit 4 2.5 weeks Using Recursion in Models & Decision Making	Unit 5 2 weeks Using Functions in Models & Decision Making	Unit 6 <sup>3 weeks</sup> Decision Making in Finance	Unit 7 2.5 weeks Networks & Graphs		
MAMDMN1. Students will extend the understanding of proportional reasoning, ratios, rates, and percents by applying them to various settings to include business, media, and consumerism. a. Use proportional reasoning to solve problems involving ratios. b. Understand and use averages, weighted averages, and indices. c. Solve problems involving large quantities that are not easily measured. d. Understand how identification numbers, such as UPCs, are created and verified.	MAMDMD1. Students will determine probability and expected value to inform everyday decision making. a. Determine conditional probabilities and probabilities of compound events to make decisions in problem situations. b. Use probabilities to make and justify decisions about risks in everyday life. c. Calculate expected value to analyze mathematical fairness, payoff, and risk.	MAMDMD2. Students will build the skills and vocabulary necessary to analyze and critique reported statistical information, summaries, and graphical displays. MAMDMD3. Students will apply statistical methods to design, conduct, and analyze statistical studies.	MAMDMD4. Students will use functions to model problem situations in both discrete and continuous relationships. a. Determine whether a problem situation involving two quantities is best modeled by a discrete (pattern identification, population growth, compound interest) or continuous (medication dosage, climate change, bone decay) relationship. b. Use linear, exponential, logistic, piecewise and sine functions to construct a model. MAMDMG1. Students will create and use two- and three-dimensional representations. MAMDMG2. Students will solve geometric problems involving inaccessible distances using basic trigonometric principles, including the Law of Sines and the Law of Cosines.	MAMDMD4. Students will use functions to model problem situations in both discrete and continuous relationships. a. Determine whether a problem situation involving two quantities is best modeled by a discrete (pattern identification, population growth, compound interest) or continuous (medication dosage, climate change, bone decay) relationship. b. Use linear, exponential, logistic, piecewise and sine functions to construct a model. MAMDMG1. Students will create and use two- and three-dimensional representations. MAMDMG2. Students will solve geometric problems involving inaccessible distances using basic trigonometric principles, including the Law of Sines and the Law of Cosines.	<ul> <li>MAMDMA3. Students will create and analyze mathematical models to make decisions related to earning, investing, spending, and borrowing money.</li> <li>a. Use exponential functions to model change in a variety of financial situations.</li> <li>b. Determine, represent, and analyze mathematical models for income, expenditures, and various types of loans and investments.</li> </ul>	<ul> <li>MAMDMA2. Students will use a variety of network models to organize data in quantitative situations, make informed decisions, and solve problems.</li> <li>a. Solve problems represented by a vertexedge graph, and find critical paths, Euler paths, and minimal spanning trees.</li> <li>b. Construct, analyze, and interpret flow charts to develop an algorithm to describe processes such as quality control procedures.</li> <li>c. Investigate the scheduling of projects using PERT.</li> <li>d. Consider problems that can be resolved by coloring graphs.</li> </ul>		

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Math of Industry & Government (MIG) Teaching & Learning Framework									
Semester 1						Semeste	r 2		
Unit 2 6 weeks	Unit 4 3 weeks	<b>Unit 5</b> 5 weeks	Unit 6 4 weeks	Unit 9 4 weeks	Unit 10 3 weeks	Unit 11 3 weeks	Unit 14 4 weeks	Unit 15 4 weeks	
Finding Optimal Solutions: Maximization	Finding Optimal Solutions	Integer Programming	Binary Programming	Critical Path Method	Critical Path	Decision Trees	Poisson Distribution	Normal Distribution	
<b>DD1b-e</b> Advanced Linear Programming	DD1b-e Advanced Linear Programming	<b>DD1a-e</b> Advanced Linear Programming	<b>DD1a-e</b> Advanced Linear Programming	DD3a-c Determine optimal paths	DD3a-c Determine optimal paths	<b>PD3d</b> Use Probabilistic Models	PD2a-c Properties of distributions for optimization & efficiency	<b>PD1a-c</b> Properties of normal distribution	
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Deterministic Decision Making: DD Probabilistic Decision Makin: PD



Math of Industry & Government (MIG) Teaching & Learning Framework									
	Block Schedule								
Unit 2 3 weeks	Unit 4 1.5 weeks	Unit 5 2.5 weeks	Unit 6 2 weeks	Unit 9 2 weeks	<b>Unit 10</b> 1.5 weeks	<b>Unit 11</b> 1.5 weeks	Unit 14 2 weeks	Unit 15 2 weeks	
Finding Optimal Solutions: Maximization	Finding Optimal Solutions	Integer Programming	Binary Programming	Critical Path Method	Critical Path	Decision Trees	Poisson Distribution	Normal Distribution	
<b>DD1b-e</b> Advanced Linear Programming	<b>DD1b-e</b> Advanced Linear Programming	<b>DD1a-e</b> Advanced Linear Programming	<b>DD1a-e</b> Advanced Linear Programming	DD3a-c Determine optimal paths	DD3a-c Determine optimal paths	PD3d Use Probabilistic Models	PD2a-c Properties of distributions for optimization & efficiency	<b>PD1a-c</b> Properties of normal distribution	
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Deterministic Decision Making: DD Probabilistic Decision Makin: PD



Statistical Reasoning Teaching & Learning Framework										
Semester 1					Semester 2					
Unit 1 3 weeks Statistical Problem Solving Process	Unit 2 3 weeks Formulatin g Questions	Unit 3 3 weeks Collecting Data	Unit 4 3 weeks The Role of Random -ness	Unit 5 3 weeks Analyzing Data	Unit 6 3 weeks Comparing Distributions	Unit 7 3 weeks Bivariate Comparisons	Unit 8 3 weeks Interpreting Results & Inference	Unit 9 3 weeks Simulations & Margin of Error	Unit 10 3 weeks Simulations & P-Value	Unit 11 3 weeks Creating Experiments & Culminating Project
MSRFQ1 Students will apply the statistical method to real- world situations; MSRCD3. Students will distinguish between the three types of study designs for collecting data (sample survey, experiment, and observational study) and will know the scope of the interpretation for each design type.	MSRFQ2. Students will identify whether the data are categorical or quantitative (numerical).	MSRCD1. Students will distinguish between a population distribution, a sample data distribution, and a sampling distribution.	MSRCD2. Students will understan d that randomne ss should be incorporat ed into a sampling or experimen tal procedure. MSRCD4. Students will distinguish between the role of randomne ss and the role of sample size with respect to using a statistic from a sample to estimate a population parameter.	MSRAD1. Students will use distributions to identify the key features of the data collected.	MSRAD2. Students will use distributions to compare two or more groups.	MSRAD3. Students will determine if an association exists between two variables (pattern or trend in bivariate data) and use values of one variable to predict values of another variable.	MSRIR1. Students will ask if the difference between two sample proportions or two sample means is due to random variation or if the difference is significant.	MSRIR2. Students will understand that when randomness is incorporated into a sampling or experimental procedure, probability provides a way to describe the 'long-run' behavior of a statistic as described by its sampling distribution.	MSRIR2. Students will understand that when randomness is incorporated into a sampling or experimental procedure, probability provides a way to describe the 'long-run' behavior of a statistic as described by its sampling distribution.	All standards for the course
These units were written to build upon concepts from prior units, so later units contain tasks that depend upon the concepts addressed in earlier units. All units will include the Mathematical Practices and indicate skills to maintain										



Statistical Reasoning Teaching & Learning Framework										
Block Schedule										
Unit 1 1.5 weeks Statistical Problem Solving Process	Unit 2 1.5 weeks Formulatin g Questions	Unit 3 1.5 weeks Collecting Data	Unit 4 1.5 weeks The Role of Random- ness	Unit 5 1.5 weeks Analyzing Data	Unit 6 1.5 weeks Comparing Distributions	Unit 7 1.5 weeks Bivariate Comparisons	Unit 8 1.5 weeks Interpreting Results & Inference	Unit 9 1.5 weeks Simulations & Margin of Error	Unit 10 1.5 weeks Simulations & P-Value	Unit 11 1.5 weeks Creating Experiments & Culminating Project
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These units were written to build upon concepts from prior units, so later units contain tasks that depend upon the concepts addressed in earlier units. All units will include the Mathematical Practices and indicate skills to maintain										

College Readiness Mathematics Teaching & Learning Framework									
		1 <sup>st</sup>	Semester		2 <sup>nd</sup> Semester				
Unit 1 <sup>3</sup> weeks Algebraic Expression S MGSE9-	Unit 2 3 weeks Equations MGSE8.EE.7	Unit 3 3 weeks Measurement & Proportional Reasoning MGSE9-	Unit 4 6 weeks Linear Functions MGSE8.EE.1	Unit 5 4 weeks Linear System of Equations	Unit 6 6 weeks Quadratic Functions MGSE9-12.A.SSE.1	Unit 7 6 weeks Exponential Functions MGSE9-12.A.SSE.1	Unit 8 5 weeks Summarizing & Interpreting Statistical Data MGSE9-12.S.ID.1		
12.N.Q.1 Units of Measure MGSE9- 12.A.SSE.1 Interpret Expressions MGSE9-12- A.SSE.2 Equivalent Expressions MGSE9- 12.A.SSE.3 Properties of Expressions MGSE9- 12.F.IF.8 Write Functions	Linear Equations MGSE9-12.A.SSE.1 Interpret Expressions MGSE9-12.A.SSE.3 Properties of Expressions MGSE9- 12.A.CED.1 Create 1- Variable Equations MGSE9- 12.A.CED.2 Create 2- Variable Equations MGSE9- 12.A.CED.3 Constraints MGSE9- 12.A.CED.4 Literal Equations MGSE9- 12.A.REI.1 Justify Solutions MGSE9- 12.A.REI.2 Rational & Radical Equations MGSE9- 12.A.REI.3 Letter coefficients	12.N.Q.1 Units of Measure MGSE9- 12.N.Q.2 Descriptive Modeling MGSE9- 12.G.GPE.4 Geometric Theorems MGSE9- 12.G.GPE.7 Distance Formula MGSE9- 12.G.GMD.1 Geometric Formulas MGSE9- 12.G.GMD.3 Volume Formulas MGSE9- 12.G.MG.2 Density MGSE9- 12.G.MG.3 Design Problems	Proportional Relationships MGSE8.EE.2 Similarity & Slope MGSE8.F.1 Understand a function MGSE8.F.2 Compare 2 functions MGSE8.F.3 Linear functions MGSE9.F.4 Construct a function MGSE9-12.A.CED.2 Create 2 Variable Equations MGSE9-12.F.IF.4 Characteristics of Functions MGSE9-12.F.IF.7 Graph Functions MGSE9-12.F.IF.7 Graph Linear & Quadratic MGSE9-12.F.IF.9 Compare function properties MGSE9-12.F.IE.1 Linear vs. exponential MGSE9-12.F.IE.3 Function observations MGSE9-12.S.ID.6 Scatter plots MGSE9-12.S.ID.6 Linear Regression MGSE9-12.S.ID.7 Interpret y=mx+b	Create 2 Variable Equations MGSE9-12.A.CED.3 Constraints MGSE9-12.A.REI.5 Elimination method MGSE9-12.A.REI.6 Graphing method MGSE9-12.A.REI.11 Solutions MGSE9-12.A.REI.12 Graph the solution MGSE9-12.F.IF.9 Compare function properties	Interpret Expressions MGSE9-12.A.SSE.1a Interpret Expression Parts MGSE9-12.A.SSE.2 Equivalent Expressions MGSE9-12.A.SSE.3 Properties of Expressions MGSE9-12.A.SSE.3a Factor Quadratic Expressions MGSE9-12.A.SSE.3b Complete the square MGSE9-12.A.CED.1 Create 1-Variable Equations MGSE9-12.A.REI.4 Solve 1-variable quadratic equations MGSE9-12.A.REI.4 Derive the quadratic formula MGSE9-12.A.REI.4b Solve quadratic equations by inspection MGSE9-12.A.REI.7 Solve system linear/quadratic MGSE9-12.F.IF.7 Solve system linear/quadratic MGSE9-12.F.IF.7 Graph Functions MGSE9-12.F.IF.7a Graph Linear/Quadratic Functions MGSE9-12.F.IF.73 Even/Odd Functions The Value of k	Interpret Expressions MGSE9-12.A.SSE.1a Interpret Expression Parts MGSE9-12.A.SSE.2 Equivalent Expressions MGSE9-12.A.SSE.3 Properties of Expressions MGSE9-12.A.SSE.3a Factor Quadratic Expressions MGSE9-12.A.SSE.3b Complete the square MGSE9-12.A.CED.2 Create 2 Variable Equations MGSE9-12.A.CED.4 Literal Equations MGSE9-12.F.IF.4 Characteristics of Functions MGSE9-12.F.IF.7 Graph Functions MGSE9-12.F.IF.7 Graph Functions MGSE9-12.F.IF.8b Interpret exponential expressions MGSE9-12.F.BF.1 Write a function MGSE9-12.F.BF.13 Explicit/recursive MGSE9-12.F.BF.2 Arithmetic/geometric MGSE9-12.F.IE.16 Growth of Function MGSE9-12.F.LE.11 Linear vs. exponential MGSE9-12.F.LE.12 Constant Rates MGSE9-12.F.LE.12 Construct Linear & exponential MGSE9-12.F.LE.3 Function observations	Dot & box plots MGSE9-12.S.ID.2 Center & Spread MGSE9-12.S.ID.3 Outliers MGSE9-12.S.ID.5 2-way tables MGSE9-12.S.ID.6 Scatter plots MGSE9-12.S.ID.6a Type of function MGSE9-12.S.ID.7 Interpret y=mx+b MGSE9-12.S.ID.8 Correlation coefficient MGSE9-12.S.ID.9 Correlation vs. causation MGSE9-12.S.ID.9 Correlation vs. causation MGSE9-12.S.IC.1 Making inferences MGSE9-12.S.IC.3 Randomization		
_						Interpret parameters			

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Grades 9-12 Key: Algebra Strand: SSE = Seeing Structure in Expressions, APR = Arithmetic with Polynomial and Rational Expressions, CED = Creating Equations, REI = Reasoning with Equations and Inequalities Functions Strand: IF = Interpreting Functions, LE = Linear and Exponential Models, BF = Building Functions, TF = Trigonometric Functions

Geometry Strand: CO = Congruence, SRT = Similarity, Right Triangles, and Trigonometry, C = Circles, GPE = Expressing Geometric Properties with Equations, GMD = Geometric Measurement and Dimension, MG = Modeling with Geometry

Statistics and Probability Strand: ID = Interpreting Categorical and Quantitative Data, IC = Making Inferences and Justifying Conclusions, CP = Conditional Probability and the Rules of Probability, MD = Using Probability to Make Decisions

College Readiness Mathematics Teaching & Learning Framework										
Block Schedule										
Unit 1 1.5 weeks	Unit 2 1.5 weeks	Unit 3 1.5 weeks Measurement	Unit 4 3 weeks	Unit 5 2 weeks	Unit 6 3 weeks	Unit 7 3 weeks	Unit 8 2.5 weeks			
Expression s	-400000	& Proportional Reasoning		Equations			Interpreting Statistical Data			
MGSE9- 12.N.Q.1 Units of Measure MGSE9- 12.A.SSE.1 Interpret Expressions MGSE9-12- A.SSE.2 Equivalent Expressions MGSE9- 12.A.SSE.3 Properties of Expressions MGSE9- 12.F.IF.8 Write Functions	MGSE8.EE.7 Linear Equations AGSE9-12.A.SSE.1 Interpret Expressions MGSE9-12.A.SSE.3 Properties of Expressions MGSE9- 12.A.CED.1 Create 1- Variable Equations MGSE9- 12.A.CED.2 Create 2- Variable Equations MGSE9- 12.A.CED.3 Constraints MGSE9- 12.A.CED.3 Constraints MGSE9- 12.A.CED.4 Literal Equations MGSE9- 12.A.CED.4 Literal Equations MGSE9- 12.A.REI.1 ustify Solutions MGSE9- 12.A.REI.2 Rational & Radical Equations MGSE9- 12.A.REI.3 Letter coefficients	MGSE9- 12.N.Q.1 Units of Measure MGSE9- 12.N.Q.2 Descriptive Modeling MGSE9- 12.G.GPE.4 Geometric Theorems MGSE9- 12.G.GPE.7 Distance Formula MGSE9- 12.G.GMD.1 Geometric Formulas MGSE9- 12.G.GMD.3 Volume Formulas MGSE9- 12.G.MG.2 Density MGSE9- 12.G.MG.3 Design Problems	MGSE8.EE.1 Proportional Relationships MGSE8.EE.2 Similarity & Slope MGSE8.F.1 Understand a function MGSE8.F.2 Compare 2 functions MGSE8.F.3 Linear functions MGSE9.F.4 Construct a function MGSE9-12.A.CED.2 Create 2 Variable Equations MGSE9-12.F.IF.4 Characteristics of Functions MGSE9-12.F.IF.7 Graph Functions MGSE9-12.F.IF.7 Graph Linear & Quadratic MGSE9-12.F.IF.9 Compare function properties MGSE9-12.F.IE.1 Linear vs. exponential MGSE9-12.F.IE.3 Function observations MGSE9-12.S.ID.6 Linear Regression MGSE9-12.S.ID.7 Interpret y=mx+b	MGSE9-12.A.CED.2 Create 2 Variable Equations MGSE9-12.A.CED.3 Constraints MGSE9-12.A.REI.5 Elimination method MGSE9-12.A.REI.6 Graphing method MGSE9-12.A.REI.11 Solutions MGSE9-12.A.REI.12 Graph the solution MGSE9-12.F.IF.9 Compare function properties	MGSE9-12.A.SSE.1 Interpret Expressions MGSE9-12.A.SSE.1a Interpret Expression Parts MGSE9-12.A.SSE.2 Equivalent Expressions MGSE9-12.A.SSE.3 Properties of Expressions MGSE9-12.A.SSE.3a Factor Quadratic Expressions MGSE9-12.A.SSE.3b Complete the square MGSE9-12.A.CED.1 Create 1-Variable Equations MGSE9-12.A.REI.4 Solve 1-variable Equadratic equations MGSE9-12.A.REI.4 Derive the quadratic formula MGSE9-12.A.REI.4b Solve quadratic equations by inspection MGSE9-12.A.REI.7 Solve system linear/quadratic MGSE9-12.F.IF.4 Characteristics of Functions MGSE9-12.F.IF.7 Graph Functions MGSE9-12.F.IF.73 Graph Linear/Quadratic Functions MGSE9-12.F.IF.73 Even/Odd Functions The Value of k	MGSE9-12.A.SSE.1 Interpret Expressions MGSE9-12.A.SSE.1a Interpret Expression Parts MGSE9-12.A.SSE.2 Equivalent Expressions MGSE9-12.A.SSE.3 Properties of Expressions MGSE9-12.A.SSE.3a Factor Quadratic Expressions MGSE9-12.A.SSE.3b Complete the square MGSE9-12.A.CED.2 Create 2 Variable Equations MGSE9-12.A.CED.4 Literal Equations MGSE9-12.F.IF.4 Characteristics of Functions MGSE9-12.F.IF.7 Graph Functions MGSE9-12.F.IF.7 Graph Exp./Log Functions MGSE9-12.F.IF.8 Write Functions MGSE9-12.F.IF.8b Interpret exponential expressions MGSE9-12.F.BF.1 Write a function MGSE9-12.F.BF.1 Explicit/recursive MGSE9-12.F.BF.1 Explicit/recursive MGSE9-12.F.IE.1 Linear vs. exponential MGSE9-12.F.LE.1 Growth of Function MGSE9-12.F.LE.1a Growth of Function MGSE9-12.F.LE.1b Constant Rates MGSE9-12.F.LE.1c Growth & decay MGSE9-12.F.LE.3 Function observations MGSE9-12.F.LE.3 Function observations	MGSE9-12.S.ID.1 Dot & box plots MGSE9-12.S.ID.2 Center & Spread MGSE9-12.S.ID.3 Outliers MGSE9-12.S.ID.5 2-way tables MGSE9-12.S.ID.6 Scatter plots MGSE9-12.S.ID.6a Type of function MGSE9-12.S.ID.7 Interpret y=mx+b MGSE9-12.S.ID.8 Correlation coefficient MGSE9-12.S.ID.9 Correlation vs. causation MGSE9-12.S.IC.1 Making inferences MGSE9-12.S.IC.3 Randomization			

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