AP Calculus Summer Packet Exercises

These exercises represent some of the more fundamental concepts that you are expected to know from previous mathematics courses as you enter AP Calculus AB. As you work on these problems, you might encounter concepts that you have forgotten. There are many resources available (Internet, books, etc.) that you can use to get help. Use the section titles as prompts for Google searches. These exercises are expected to be completed and are due the first day of class.

Average Rate of Change and Linear Functions

1.

x	0	1	2	3	4	5	6
$f(x)$	–10	–8	–1	0	5	7	2

Using the table of values given above, find the average rate of change of f on the interval $[2, 5]$ and write an equation for the secant line passing through the corresponding points.

2. Find the equation of the line that passes through (2, –1) and is perpendicular to the line $2x - 3y = 5$.

Graphing General Curves

3. Create a "Parent Functions Cheat Sheet" by graphing all of the following parent functions on a single piece of graph paper:

a) $f(x) = x$ b) $f(x) = x^2$ c) $f(x) = x^3$ d) $f(x) = \sqrt{x}$

e) $f(x) = \sqrt[3]{x}$ f) $f(x) = |x|$ g) $f(x) = \frac{1}{x}$ h) $f(x) = \frac{1}{x^2}$

i) $f(x) = e^x$ j) $f(x) = \ln x$ k) $f(x) = \sin x$ l) $f(x) = \cos x$

m) $f(x) = \tan x$ n) $f(x) = \sec x$ o) $f(x) = \sin^{-1} x$ p) $f(x) = \tan^{-1} x$

* You may use a graphing calculator, if needed.

4. Using your "Parent Graph Cheat Sheet," graph the following without a calculator:

a) $y = (x - 3)^2 + 2$ b) $y = (x + 1)^3$ c) $y = 2 - \sqrt{x}$ d) $y = e^{-x}$

e) $y = \ln(x - 1)$ f) $y = \sqrt[3]{1 - x}$ g) $y = 2\sin x$ h) $y = \cos \pi x$

i) $y = \tan x + 1$ j) $y = 2|x - 3|$ k) $y = \frac{1}{1 + x}$

5. Graph $y = \frac{2x}{x - 4}$. Label x- and y-intercepts and all vertical and horizontal asymptotes.

Piecewise Functions

6. Suppose $f(x)=\begin{cases} x-1, \text{ if } x<2 \\ \sqrt{x-1}, \text{ if } x>2 \end{cases}$. Answer the following:

a) Calculate $f(-3)$ b) Calculate $f(2)$

c) Calculate $f(10)$ d) Graph $f(x)$

Composite Functions

7. If $f(x)=\dfrac{x}{x-1}$ and $g(x)=\dfrac{1}{x-1}$, find $(f\circ g)(x)$ and $(g\circ f)(x)$.

8. Given that $h(x)=\tan^{-1}(1-\pi x)$ is a composite function of the form $h(x)=f(g(x))$, find *f* and *g*.

Solving Equations and Inequalities

9. Solve the following.

a) $4t^3-12t^2+8t=0$ b) $3\sqrt{x-2}-8=8$ c) $2\ln 3x=4$

d) $\dfrac{x-5}{3-x}\geq 0$ e) $\left|2-\dfrac{x}{3}\right|<5$ f) $4e^{2x}=5$

g) $(x-4)-5(x-4)^{1/2}=6$ h) $2\sin^2 x=\sin x+1;\ 0\leq x\leq 2\pi$

10. Solve the linear system described as $\begin{cases} 5a+3b=9 \\ 2a-4b=14 \end{cases}$.

Simplifying Expressions

11. Factor the following expressions:

a) $3x^3+192$ b) $2x^3-11x^2+12x+9$

c) $2x^{5/4}+x^{3/4}-15x^{1/4}$ d) $9x^2-3x-2$

12. Simplify the following expressions:

a) $2\ln(x-3)+\ln(x+2)-6\ln x$ b) $\dfrac{\dfrac{2}{x}-3}{1-\dfrac{1}{x-1}}$

c) $x(1-2x)^{-3/2}+(1-2x)^{1/2}$ d) $\dfrac{e^{3x}-e^{x}}{e^{4x}}$

13. Simplify the expression $\dfrac{x^2}{1+x^2}$ using the following two methods: (Yes, you have to **think** about part b!)
a) long division b) "adding zero"

14. Find the conjugate for the following rational radical expressions. Then, use it to simplify the expression. (Yes, we **do** sometimes conjugate the numerator in calculus!)
a) $\dfrac{x}{\sqrt{x}-\sqrt{x-1}}$ b) $\dfrac{\sqrt{3+x}-\sqrt{3}}{x}$

Distance Formula

15. Find the distance between $\left(\frac{1}{2}, -7\right)$ and $(-3, 4)$.

Patterns, Sequences, and Factorials (Oh, My!)

16. Find the first four terms of the sequence $a_n = n^3 - 1$.

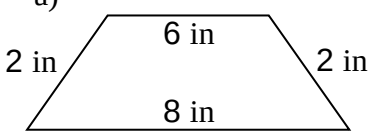
17. Find the 99th term of the sequence $\{a_n\}_{n=0}^{\infty} = \{-1, 0, 4, 1, -1, 0, 4, 1\ldots\}$. (Assume that the sequence repeats according to the alternating four-term pattern above as *n* increases.)

18. The symbol $n!$ is expressed as "*n* factorial." Although a definition is rather hard to provide, an example of a factorial is as follows: $6! = 6\cdot5\cdot4\cdot3\cdot2\cdot1 = 720$. Simplify the following:
a) $4!$ b) $\dfrac{5!}{6!}$ c) $\dfrac{3!\cdot7!}{5!}$ d) $\dfrac{n!(n-1)!}{(n+1)!}$

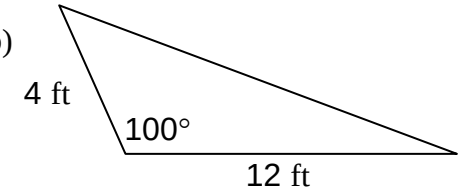
Geometry

19. Find the area and perimeter (or circumference) of each figure.

a)



b)



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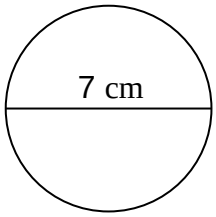
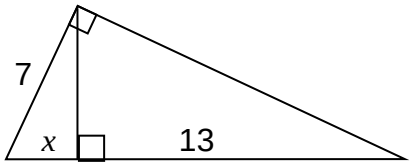


20. Find *x*.



Trigonometry

21. *Example*: Find all of the zeros for the function $y = \cos x$.
Solution: Since there are an infinite number of zeros for *y*, we must give our answer in terms of the integer *n*. Since *y* is equal to zero for "all odd $\pi/2$," we write our solution as $x = (2n+1)\frac{\pi}{2}$, taking the convention that an odd number is simply an even number plus one (minus one would have worked, too).
Based on the above, find all solutions for zeros and/or undefined values for the following functions (express your answers as domain values):
a) $y = \sin x$ b) $y = \sec x$ c) $y = \tan x$

22. Find the following: (You may need to construct or find a unit circle first!)
a) $\sin\frac{7\pi}{6}$ b) $\cos 120°$ c) $\tan\frac{\pi}{2}$
d) $\csc 60°$ e) $\sec\left(-\frac{2\pi}{3}\right)$ f) $\cot(-135°)$

23. Simplify the following trigonometric expressions: (You know that you **love** trig identities!!)
a) $4\sin 2x \cos 2x$ b) $1 - \sec^2 x$ c) $\frac{1+\cos 2x}{2}$
d) $\cos^2 x - \sin^2 x$ e) $\cos^2 x + \sin^2 x$

AP Calculus Summer Packet Answer Sheet

Write you answers in the spaces provided.

1.

2.

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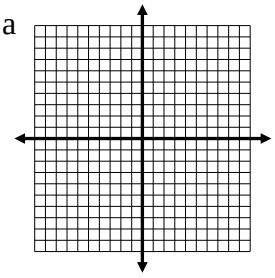
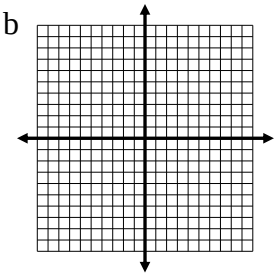
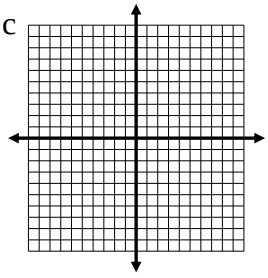
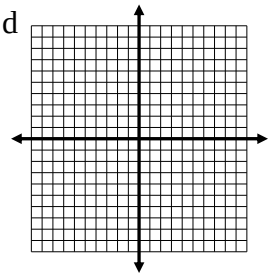
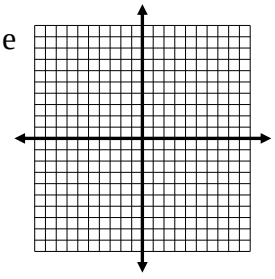
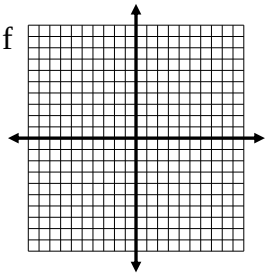
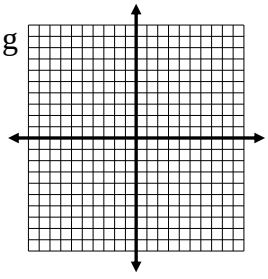
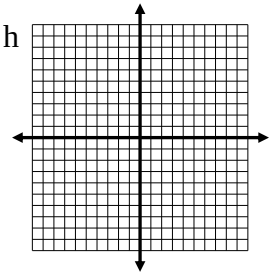
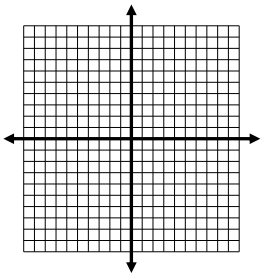
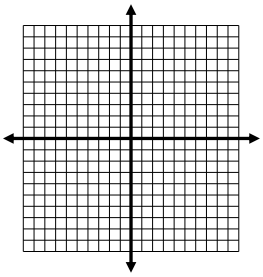
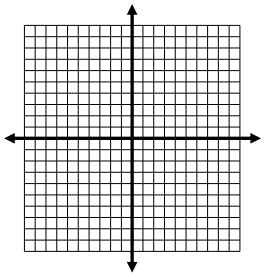
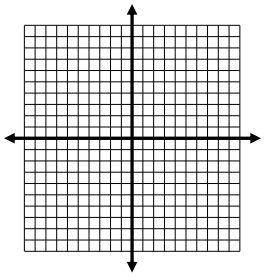
3. Use your own graph paper.

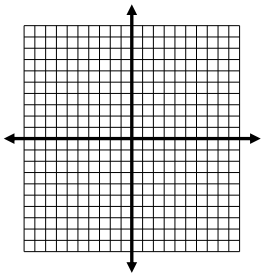
4. a  b  c  d 

e  f  g  h 

i  j  k 

5. 

6. a _____ b _____

c _____ d 

7.

8.

9. a _____ b _____ c _____

d _____ e _____ f _____

g _____ h _____

10.

11. a _____ b _____

c _____ d _____

12. a _____ b _____

c _____ d _____

13. a b

14. a _____ b _____

15.

16.

17.

18. a _____ b _____

c _____ d _____

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19. a _____ b _____

c _____

20.

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21. a _____ b _____ c _____

22. a _____ b _____ c _____

d _____ e _____ f _____

23. a _____ b _____ c _____

d _____ e _____